MODUL PENTAKSIRAN BILIK DARJAH

KUASAI PBD

TINGKATAN 5 KSSM

KHAS UNTUK GURU

MATEMATIK TAMBAHAN
ADDITIONAL MATHEMATICS

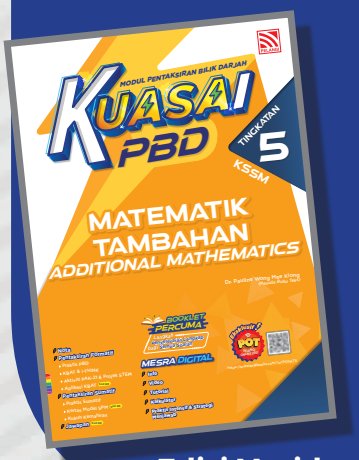
Mempermudah Pentaksiran Bilik Darjah (PBD)

Melancarkan Pentaksiran Formatif dan Sumatif

Menyokong Pembelajaran dan Pemudahcaraan (PdPc) Mesra Digital

Meningkatkan Tahap Penguasaan Murid

Edisi Guru



Edisi Murid

PAKEJ PERCUMA UNTUK KEMUDAHAN GURU

EDISI GURU

VERSI CETAK

- Nota
- Praktis PBD
- Praktis Sumatif
- Kukuh Kemahiran
- Aplikasi KBAT
- Kertas Model SPM
- Jawapan
- Bahan Digital

RESOS DIGITAL GURU

ePelangi+

Pelbagai bahan digital sokongan PdPc yang disediakan khas untuk guru di platform ePelangi+

EG-i + BAHAN SOKONGAN PdPc EKSTRA!

EDISI GURU (versi cetak)

PANDUAN PENGGUNAAN

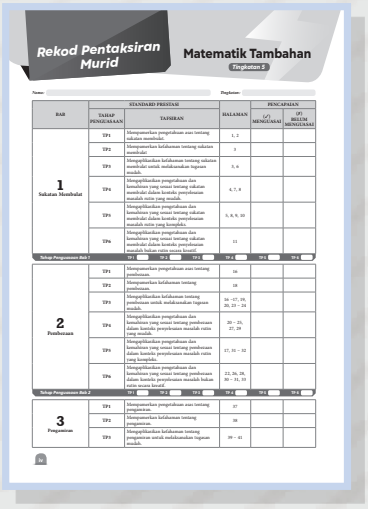
A Kandungan

Kandungan disertakan rujukan bahan-bahan digital sokongan dalam buku.


KANDUNGAN

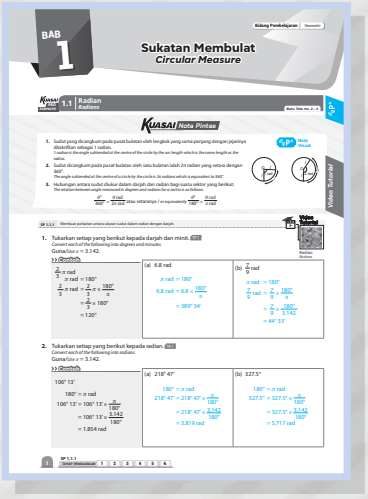
B Rekod Pentaksiran Murid

Jadual untuk catatan prestasi Tahap Penguasaan murid.

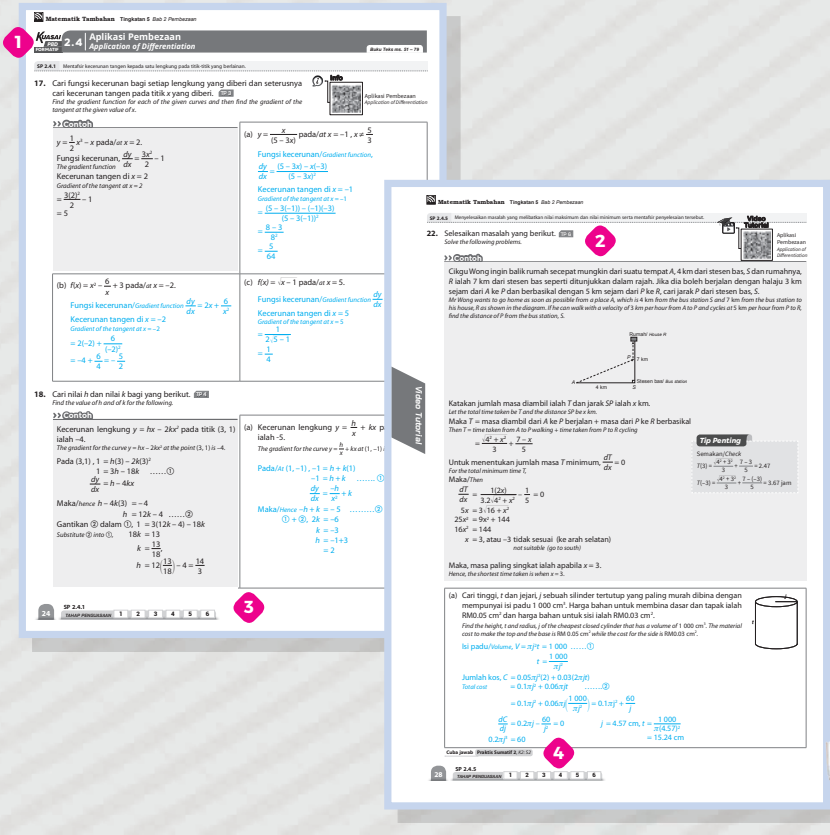

Rekod Pentaksiran Murid — Matematik Tambahan, Tingkatan 5

C Nota

Nota ringkas di halaman permulaan setiap subtopik.


BAB 1 Sukatan Membulat — Circular Measure

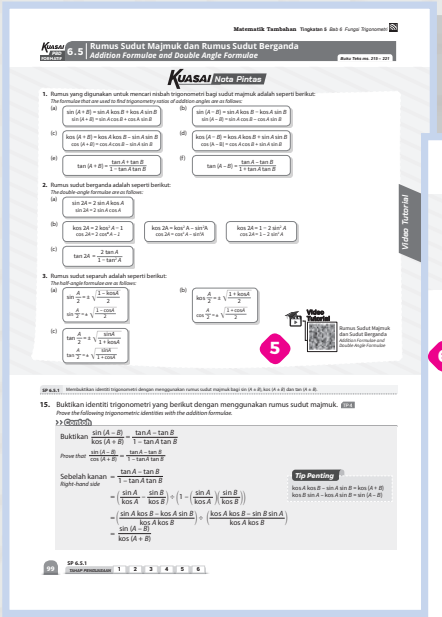
D Praktis PBD » Pentaksiran Berterusan

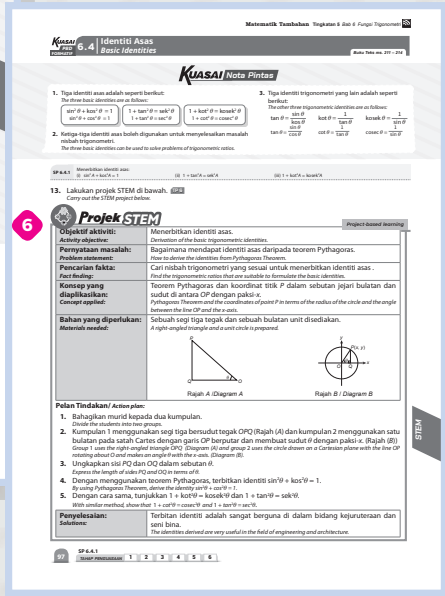

Matematik Tambahan Tingkatan 5 Bab 2 Pembezaan — 2.4 Aplikasi Pembezaan / Application of Differentiation

1. Soalan latihan formatif dirangka jelas mengikut Standard Kandungan (SK) dan Standard Pembelajaran (SP) sejajar dengan halaman buku teks.
2. Soalan dikriteriakan mengikut 6 Tahap Penguasaan (TP). Soalan Kemahiran Berfikir Aras Tinggi (KBAT) dikenal pasti.
3. Tahap penguasaan murid boleh dinilai di akhir setiap halaman.
4. **Cuba Jawab** merujuk silang soalan kepada Praktis Sumatif (soalan berbentuk penilaian) di akhir bab untuk menguji tahap kefahaman murid.

5 Bahan digital lain seperti **Kalkulator**, **Info**, **Video** dan **Video Tutorial** disediakan untuk meningkatkan keseronokan pembelajaran Matematik.

6 Aktiviti seperti Projek STEM dan PAK-21 disertakan untuk menyempurnakan PdPc.





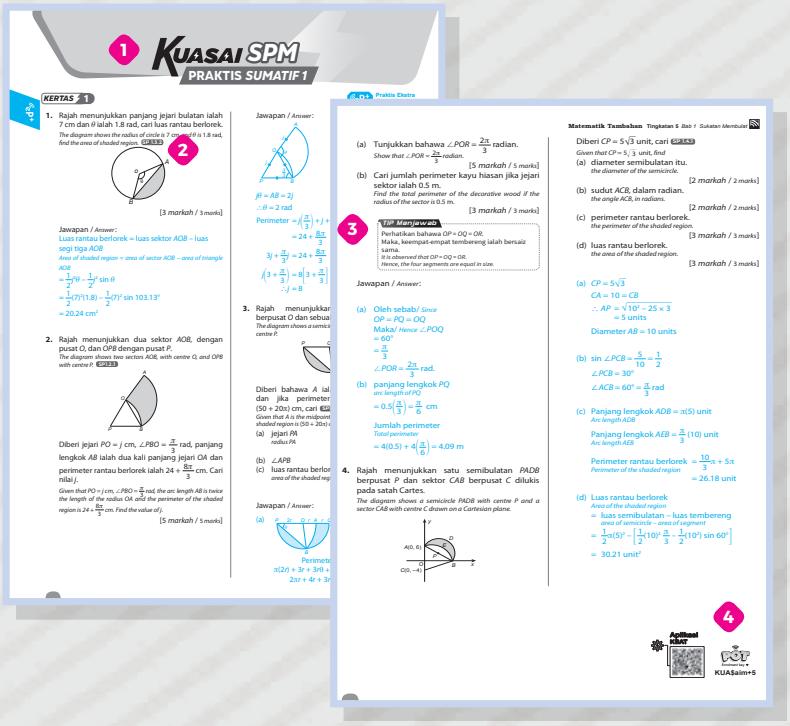
E Praktis Sumatif

1 Soalan latihan pada akhir bab ini memberikan pendedahan awal kepada murid untuk menguasai format SPM sebenar.

2 Soalan mencakupi pelbagai SP.

3 **Tip Menjawab** memberikan maklumat tambahan kepada murid supaya mereka dapat menjawab soalan aras kesukaran tinggi dengan yakinnya.

4 **POT (*Pelangi Online Test*)** menggalakkan murid untuk membanyakkan latihan soalan objektif mengikut topik secara dalam talian. Satu set Pentaksiran SPM juga disediakan. Ikon POT berserta ***Enrolment Key*** boleh didapati di halaman akhir setiap bab.



CARA MENGAKSES POT (Portal Ujian Soalan Objektif)

Eksklusif! POT PELANGI ONLINE TEST

1 Imbas kod QR atau layari *link* di kulit depan buku untuk *Create new account*.

2 Semak e-mel untuk mengaktifkan akaun.

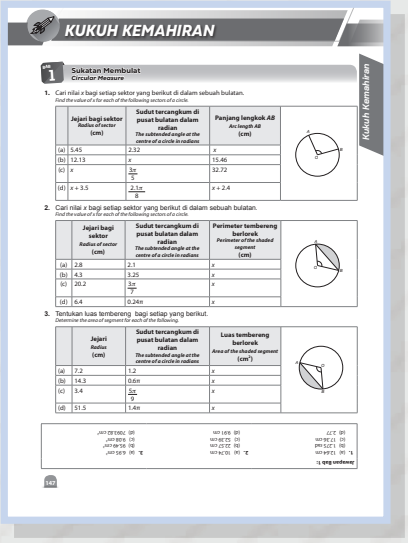
3 *Log in* ke akaun anda.

4 Masukkan *Enrolment Key*.

5 Mulakan ujian!

F Kukuh Kemahiran

Soalan latihan bagi mengukuhkan kemahiran murid yang merangkumi setiap bab Matematik Tambahan Tingkatan 5.



KUKUH KEMAHIRAN

Kukuh Kemahiran

1 Sukatan Membulat
Circular Measure

1. Cari nilai x bagi setiap sektor yang berikut di dalam sebuah bulatan.
Find the value of x for each of the following sectors of a circle.

	Jejari bagi sektor *Radius of sector* (cm)	Sudut tercangkum di pusat bulatan dalam radian *The subtended angle at the centre of a circle in radians*	Panjang lengkok AB *Arc length AB* (cm)
(a)	5.45	2.32	x
(b)	12.13	x	15.46
(c)	x	$\frac{3\pi}{5}$	32.72
(d)	$x + 3.5$	$\frac{2.1\pi}{8}$	$x + 2.4$

2. Cari nilai x bagi setiap sektor yang berikut di dalam sebuah bulatan.
Find the value of x for each of the following sectors of a circle.

	Jejari bagi sektor *Radius of sector* (cm)	Sudut tercangkum di pusat bulatan dalam radian *The subtended angle at the centre of a circle in radians*	Perimeter tembereng berlorek *Perimeter of the shaded segment* (cm)
(a)	2.8	2.1	x
(b)	4.3	3.25	x
(c)	20.2	$\frac{3\pi}{7}$	x
(d)	6.4	0.24π	x

3. Tentukan luas tembereng bagi setiap yang berikut.
Determine the area of segment for each of the following.

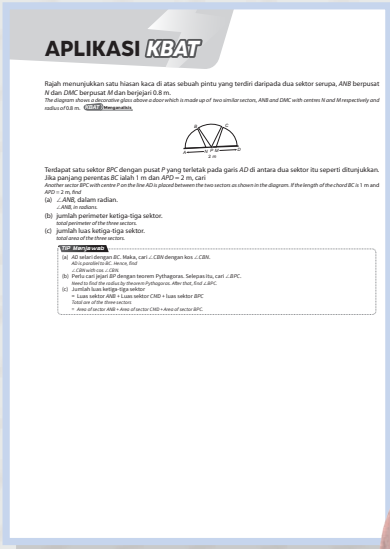
	Jejari *Radius* (cm)	Sudut tercangkum di pusat bulatan dalam radian *The subtended angle at the centre of a circle in radians*	Luas tembereng berlorek *Area of the shaded segment* (cm^2)
(a)	7.2	1.2	x
(b)	14.3	0.6π	x
(c)	3.4	$\frac{2\pi}{3}$	x
(d)	51.5	1.4π	x

Jawapan bab 1:
1. (a) 6.85 cm (b) 7.05 cm
2. (a) 2.4 (b) 5.25 cm
3. (a) 3.77 (b) 5.39 cm (c) 10.68 cm² (d) 1.25 cm²

142

G Aplikasi KBAT

Soalan latihan berfokus KBAT (dalam kod QR) di akhir halaman Praktis Sumatif ini merangsang pemikiran yang berstruktur dan berfokus dalam kalangan murid.



APLIKASI KBAT

Rajah menunjukkan satu hiasan kaca di atas sebuah pintu yang terdiri daripada dua sektor serupa, ANB berpusat N dan DMC berpusat M dan berjejari 0.8 m.
The diagram shows a decorative glass above a door which is made up of two similar sectors, ANB and DMC with centres N and M respectively and radius of 0.8 m. **KBAT Mengaplikasi.**

Terdapat satu sektor BPC dengan pusat P yang terletak pada garis AD di antara dua sektor itu seperti ditunjukkan. Jika panjang perentas BC ialah 1 m dan $APD = 2$ m, cari
Area of sector BPC with centre P on the line AD is placed between the two sectors as shown in the diagram. If the length of the chord BC is 1 m and $APD = 2$ m, find

(a) $\angle ANB$, dalam radian.
$\angle ANB$, in radians.

(b) jumlah perimeter ketiga-tiga sektor.
total perimeter of the three sectors.

(c) jumlah luas ketiga-tiga sektor.
total area of the three sectors.

TIP Mengawal

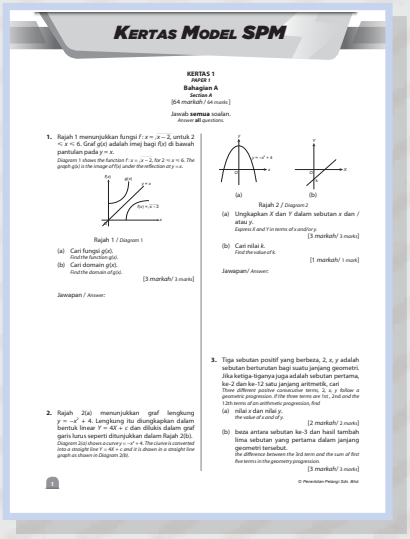
(a) AD selari dengan BC. Maka, cari $\angle CBN$ dengan kos $\angle CBN$.
AD is parallel to BC. Hence, find
$\angle CBN$ with cos $\angle CBN$

(b) Perlu cari jejari BP dengan teorem Pythagoras. Selepas itu, cari $\angle BPC$.
Need to find the radius by theorem Pythagoras. After that, find $\angle BPC$.

(c) Jumlah luas ketiga-tiga sektor
Total area of the three sectors
= Luas sektor BPC + luas sektor CMD + luas sektor BPC
= Area of sector BPC + Area of sector CMD + Area of sector BPC.

H Kertas Model SPM » Pentaksiran Sumatif

Soalan penilaian (dalam kod QR) yang mengikut format SPM sebenar.



KERTAS MODEL SPM

KERTAS 1
PAPER 1
Bahagian A
Section A
[64 markah / 64 marks]
Jawab **semua** soalan.
*Answer **all** questions.*

1. Rajah 1 menunjukkan fungsi $f : x = x - 2$, untuk $2 \le x < 6$. Graf $g(x)$ adalah imej bagi $f(x)$ di bawah pantulan pada $y = x$.
Diagram 1 shows the function $f : x = x - 2$, for $2 \le x < 6$. The graph $g(x)$ is the image of $f(x)$ under the reflection at $y = x$.

Rajah 1 / *Diagram 1*

(a) Cari fungsi $g(x)$.
Find the function $g(x)$.
(b) Cari domain $g(x)$.
Find the domain of $g(x)$.

[3 markah/ 3 marks]

Jawapan / *Answer:*

2. Rajah 2(a) menunjukkan graf lengkung $y = -x^2 + 4$. Lengkung itu diungkapkan dalam bentuk linear $Y = 4X + c$ dan diluluskan dalam graf garis lurus seperti ditunjukkan dalam Rajah 2(b).
Diagram 2(a) shows a curve $y = -x^2 + 4$. The curve is converted into a straight line $Y = 4X + c$ and it is shown in a straight line graph as shown in Diagram 2(b).

Rajah 2 / *Diagram 2*

(a) Ungkapkan X dan Y dalam sebutan x dan / atau y.
Express X and Y in terms of x and/or y.
[3 markah/ 3 marks]

(b) Cari nilai k.
Find the value of k.
[1 markah/ 1 mark]

Jawapan/ *Answer:*

3. Tiga sebutan positif yang berbeza, $2, x, y$ adalah sebutan berturutan bagi suatu janjang geometri. Jika ketiga-tiganya juga adalah sebutan pertama, ke-2 dan ke-12 satu janjang aritmetik, cari
Three different positive consecutive terms, $2, x, y$ follow a geometric progression. If the three terms are 1st , 2nd and the 12th terms of an arithmetic progression, find

(a) nilai x dan nilai y.
the value of x and of y.
[2 markah/ 2 marks]

(b) beza antara sebutan ke-3 dan hasil tambah lima sebutan yang pertama dalam janjang geometri tersebut.
the difference between the 3rd term and the sum of first five terms in the geometry progression.
[3 markah/ 3 marks]

1

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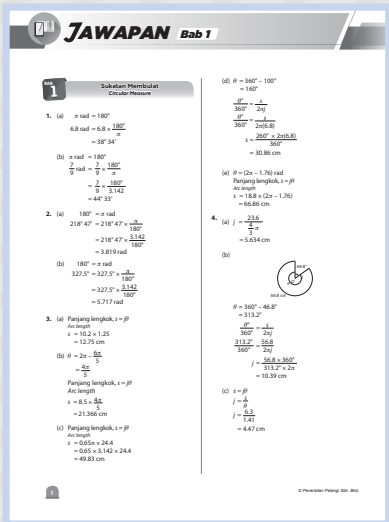
I Jawapan

Jawapan keseluruhan buku (dalam kod QR) disediakan di halaman Kandungan.



JAWAPAN Bab 1

1 Sukatan Membulat
Circular Measure

1. (a) x rad = 180°
$6.8 \text{ rad} = 6.8 \times \frac{180°}{\pi}$
$= 389.73°$
$= 389° 44'$

(b) x rad = 180°
$\frac{7}{9} \text{ rad} = \frac{7}{9} \times \frac{180°}{\pi}$
$= \frac{7}{9} \times \frac{180°}{3.142}$
$= 44° 33'$

2. (a) 180° = x rad
$218° 47' = 218° 47' \times \frac{\pi}{180°}$
$= 218° 47' \times \frac{3.142}{180°}$
$= 3.819 \text{ rad}$

(b) 180° = π rad
$327.5° = 327.5° \times \frac{\pi}{180°}$
$= 327.5° \times \frac{3.142}{180°}$
$= 5.717 \text{ rad}$

3. (a) Panjang lengkok, $s = j\theta$
Arc length
$s = 10.2 \times 1.25$
$= 12.75 \text{ cm}$

(b) $\theta = 2\pi - \frac{6\pi}{5}$
$= \frac{4\pi}{5}$
Panjang lengkok, $s = j\theta$
Arc length
$s = 8.5 \times \frac{4\pi}{5}$
$= 21.366 \text{ cm}$

(c) Panjang lengkok, $s = j\theta$
Arc length
$s = 0.65\pi \times 24.4$
$= 0.65 \times 3.142 \times 24.4$
$= 49.83 \text{ cm}$

(d) $\theta = 360° - 100°$
$= 160°$
$\frac{\theta°}{360°} = \frac{s}{2\pi j}$
$\frac{\theta°}{360°} = \frac{s}{2\pi(6.8)}$
$s = \frac{360° \times 2\pi(6.8)}{360°}$
$= 30.86 \text{ cm}$

(a) $\theta = (2\pi - 1.76) \text{ rad}$
Panjang lengkok, $s = j\theta$
Arc length
$s = 18.8 \times (2\pi - 1.76)$
$= 66.86 \text{ cm}$

4. (a) $j = \frac{23.6}{\frac{4}{3}\pi}$
$= 5.634 \text{ cm}$

(b)

$\theta = 360° - 46.8°$
$= 313.2°$
$\frac{\theta°}{360°} = \frac{s}{2\pi j}$
$\frac{313.2°}{360°} = \frac{56.8}{2\pi j}$
$j = \frac{56.8 \times 360°}{313.2° \times 2\pi}$
$= 10.39 \text{ cm}$

(c) $s = j\theta$
$j = \frac{s}{\theta}$
$j = \frac{6.3}{1.41}$
$= 4.47 \text{ cm}$

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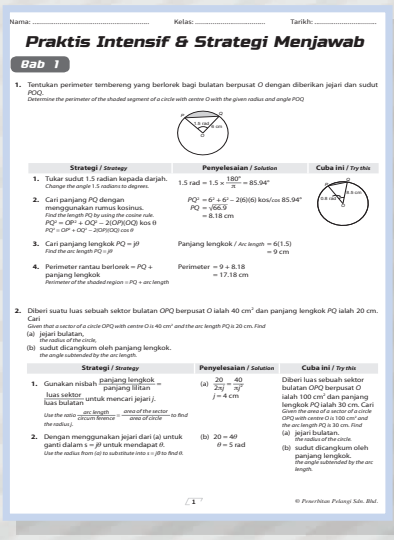
J Praktis Intensif & Strategi Menjawab

Pelbagai bentuk soalan (dalam kod QR) dengan strategi menjawab sebagai latihan kendiri untuk murid.



Nama: ..... Kelas: ..... Tarikh: .....

Praktis Intensif & Strategi Menjawab

Bab 1

1. Tentukan perimeter tembereng yang berlorek bagi bulatan berpusat O dengan diberikan jejari dan sudut POQ.
Determine the perimeter of the shaded segment of a circle with centre O with the given radius and angle POQ

Strategi / *Strategy*	Penyelesaian / *Solution*	Cuba ini / *Try this*
1. Tukar sudut 1.5 radian kepada darjah. *Change the angle 1.5 radians to degrees.*	$1.5 \text{ rad} = 1.5 \times \frac{180°}{\pi} = 85.94°$	
2. Cari panjang PQ dengan menggunakan rumus kosinus. *Find the length PQ by using the cosine rule.* $PQ^2 = OP^2 + OQ^2 - 2(OP)(OQ) \cos \theta$ $PQ^2 = OP^2 + OQ^2 - 2(OP)(OQ) \cos \theta$	$PQ^2 = 6^2 + 6^2 - 2(6)(6) \cos 85.94°$ $PQ = \sqrt{66.5}$ $= 8.18 \text{ cm}$	
3. Cari panjang lengkok $PQ = j\theta$ *Find the arc length $PQ = j\theta$*	Panjang lengkok / *Arc length* $= 6(1.5)$ $= 9 \text{ cm}$	
4. Perimeter rantau berlorek = PQ + panjang lengkok *Perimeter of the shaded region = PQ + arc length*	Perimeter $= 9 + 8.18$ $= 17.18 \text{ cm}$	

2. Diberi suatu luas sebuah sektor bulatan OPQ berpusat O ialah 40 cm^2 dan panjang lengkok PQ ialah 20 cm. Cari
Given that a sector of a circle OPQ with centre O is 40 cm^2 and the arc length PQ is 20 cm. Find
(a) jejari bulatan.
the radius of the circle.
(b) sudut dicangkum oleh panjang lengkok.
the angle subtended by the arc length.

Strategi / *Strategy*	Penyelesaian / *Solution*	Cuba ini / *Try this*
1. Gunakan nisbah $\frac{\text{panjang lengkok}}{\text{luas sektor}} = \frac{\text{panjang lilitan}}{\text{luas bulatan}}$ untuk mencari jejari j. *Use the ratio $\frac{\text{arc length}}{\text{circum ference}} = \frac{\text{area of the sector}}{\text{area of circle}}$ to find the radius j.*	(a) $\frac{20}{2\pi j} = \frac{40}{\pi j^2}$ $j = 4 \text{ cm}$	Diberi luas sebuah sektor bulatan OPQ berpusat O ialah 100 cm^2 dan panjang lengkok PQ ialah 30 cm. Cari *Given the area of a sector of a circle OPQ with centre O is 100 cm^2 and the arc length PQ is 30 cm. Find* (a) jejari bulatan. *the radius of the circle.* (b) sudut dicangkum oleh panjang lengkok. *the angle subtended by the arc length.*
2. Dengan menggunakan jejari dari (a) untuk ganti dalam $s = j\theta$ untuk mendapat θ. *Use the radius from (a) to substitute into $s = j\theta$ to find θ.*	(b) $20 = 4\theta$ $\theta = 5 \text{ rad}$	

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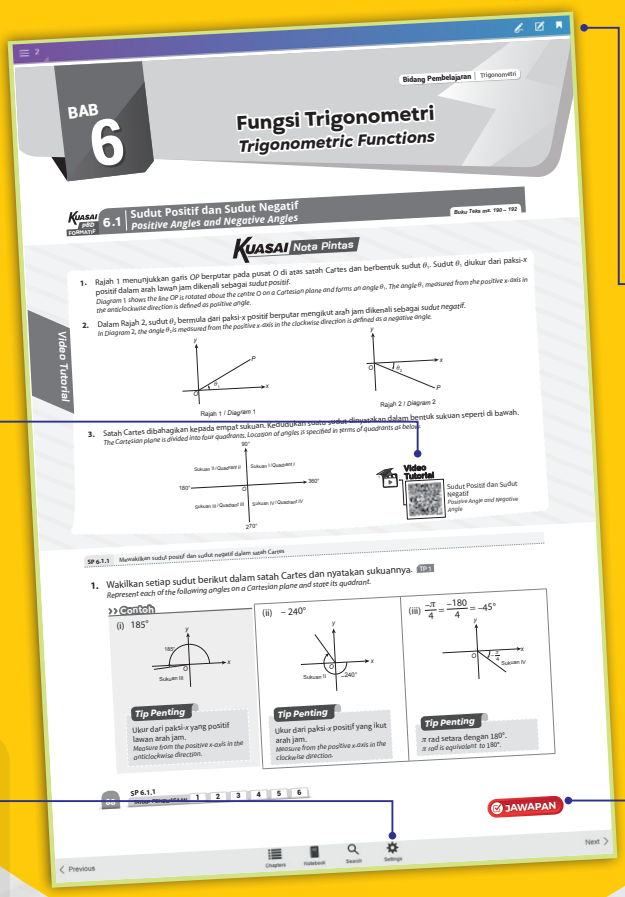
RESOS DIGITAL GURU ePelangi+

Di platform ePelangi+ , guru yang menerima guna (*adoption*) siri Kuasai PBD KSSM diberi akses kepada EG-i dan bahan sokongan ekstra PdPc untuk tempoh satu tahun:

1 Apakah itu EG-i ?

EG-i merupakan versi digital dan interaktif Edisi Guru Kuasai PBD secara dalam talian. Versi ini akan dapat mengoptimumkan penggunaan teknologi dalam pengajaran, memaksimumkan kesan PdPc, dan membangunkan suasana pembelajaran yang menyeronokkan serta responsif dalam kalangan murid.

Halaman Contoh EG-i



Klik Kod QR untuk mengakses bahan dalam kod QR seperti Info, Kalkulator, Video, Video Tutorial & Aplikasi KBAT.

Pilih paparan halaman (*single/double page*) dan bahasa antara muka melalui *Setting*.

Alat sokongan lain:

- Pen
- Sticky Note
- Unit Converter
- Ruler
- Calculator
- Bookmark

Klik butang JAWAPAN untuk memaparkan atau melenyapkan jawapan (*hidden*) semasa penyampaian PdPc.

2 BAHAN SOKONGAN PdPc EKSTRA!

Bahan-bahan pengajaran dan latihan di platform ePelangi+ boleh dimuat turun atau dimainkan terus.

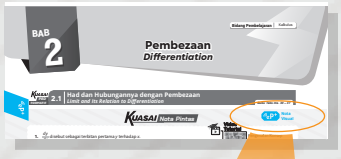
- e-RPH (Microsoft Word)
- Edisi Guru pdf
- Nota Visual
- Praktis Ekstra Sumatif
- PowerPoint Interaktif
- Simulasi

Boleh dimuat turun
Boleh dimainkan

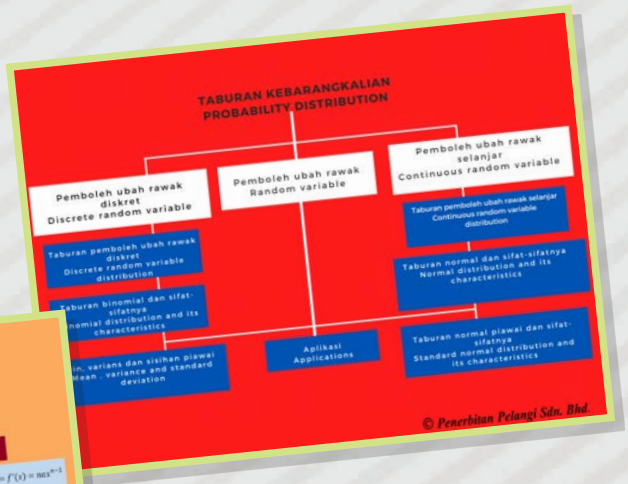
Bahan sokongan PdPc ekstra yang sesuai dicadangkan pada halaman atau bahagian tertentu Edisi Guru melalui penandaan ikon ePP+ .

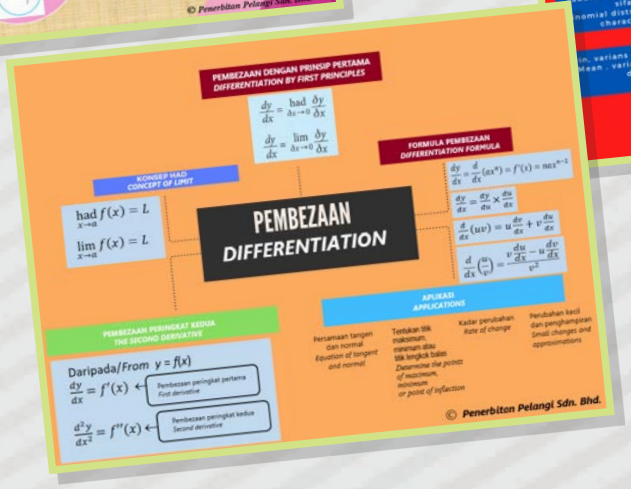
CONTOH HALAMAN EDISI GURU DENGAN CADANGAN BAHAN SOKONGAN PDPC EKSTRA

Nota Visual
Nota ringkas berwarna dalam persembahan grafik.



ePP+ Nota Visual

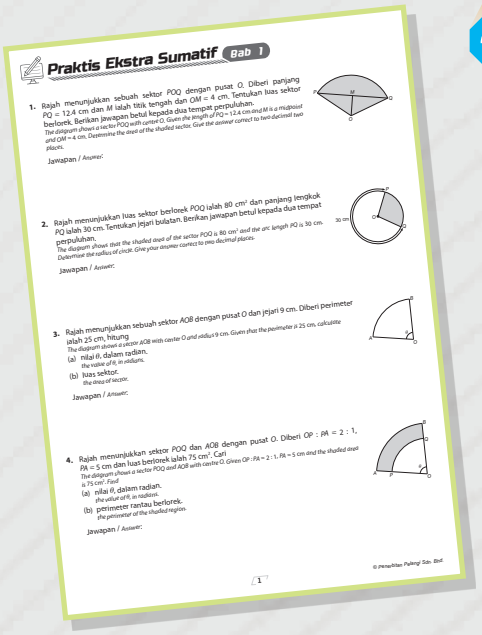




Praktis Ekstra Sumatif

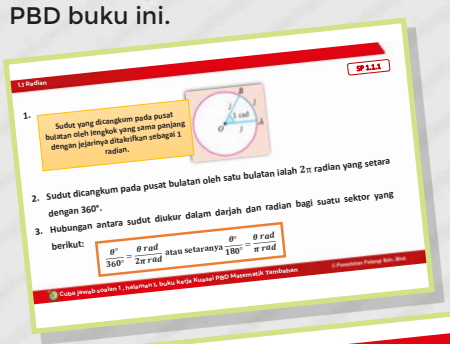
Soalan latihan tambahan mengikut bab.

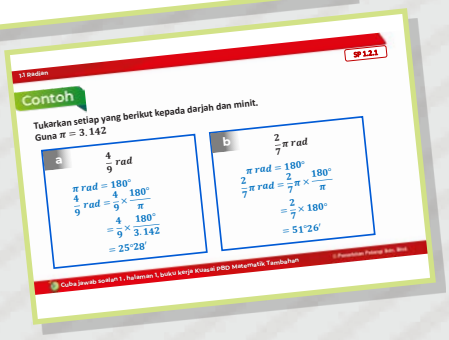
ePP+ Praktis Ekstra Sumatif



PowerPoint Interaktif

Slaid Pengajaran PowerPoint untuk memesrakan penyampaian PdPc guru sejajar dengan aktiviti modul PBD buku ini.





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LANGKAH 3

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Rekod Pentaksiran Murid

Matematik Tambahan

Tingkatan 5

Nama: _____ *Tingkatan:* _____

BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) MENGUASAI	(✗) BELUM MENGUASAI
1 Sukatan Membulat	TP1	Mempamerkan pengetahuan asas tentang sukatan membulat.	1, 2		
	TP2	Mempamerkan kefahaman tentang sukatan membulat	3		
	TP3	Mengaplikasikan kefahaman tentang sukatan membulat untuk melaksanakan tugasan mudah.	3, 6		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah rutin yang mudah.	4, 7, 8		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah rutin yang kompleks.	5, 8, 9, 10		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah bukan rutin secara kreatif.	11		
Tahap Penguasaan Bab 1	TP 1 ☐	TP 2 ☐ TP 3 ☐	TP 4 ☐	TP 5 ☐	TP 6 ☐
2 Pembezaan	TP1	Mempamerkan pengetahuan asas tentang pembezaan.	16		
	TP2	Mempamerkan kefahaman tentang pembezaan.	18		
	TP3	Mengaplikasikan kefahaman tentang pembezaan untuk melaksanakan tugasan mudah.	16 –17, 19, 20, 23 – 24		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah rutin yang mudah.	20 – 25, 27, 29		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah rutin yang kompleks.	17, 31 – 32		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah bukan rutin secara kreatif.	22, 26, 28, 30 – 31, 33		
Tahap Penguasaan Bab 2	TP 1 ☐	TP 2 ☐ TP 3 ☐	TP 4 ☐	TP 5 ☐	TP 6 ☐
3 Pengamiran	TP1	Mempamerkan pengetahuan asas tentang pengamiran.	37		
	TP2	Mempamerkan kefahaman tentang pengamiran.	38		
	TP3	Mengaplikasikan kefahaman tentang pengamiran untuk melaksanakan tugasan mudah.	39 – 41		

BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) MENGUASAI	(✗) BELUM MENGUASAI
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengamiran dalam konteks penyelesaian masalah rutin yang mudah.	41 – 46		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengamiran dalam konteks penyelesaian masalah rutin yang kompleks.	47		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengamiran dalam konteks penyelesaian masalah bukan rutin secara kreatif.	46, 48 - 49		
Tahap Penguasaan Bab 3		**TP 1** **TP 2** **TP 3**	**TP 4**	**TP 5**	**TP 6**
4 Pilih Atur Dan Gabungan	**TP1**	Mempamerkan pengetahuan asas tentang pilih atur dan gabungan.	54 – 55		
	TP2	Mempamerkan kefahaman tentang pilih atur dan gabungan.	55 – 56, 63		
	TP3	Mengaplikasikan kefahaman tentang pilih atur dan gabungan untuk melaksanakan tugasan mudah.	57 – 58		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pilih atur dan gabungan dalam konteks penyelesaian masalah rutin yang mudah.	55, 58 – 59		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pilih atur dan gabungan dalam konteks penyelesaian masalah rutin yang kompleks.	59 – 60, 63 – 64		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pilih atur dan gabungan dalam konteks penyelesaian masalah bukan rutin secara kreatif.	61 – 64		
Tahap Penguasaan Bab 4		**TP 1** **TP 2** **TP 3**	**TP 4**	**TP 5**	**TP 6**
5 Taburan Kebarangkalian	**TP1**	Mempamerkan pengetahuan asas tentang pemboleh ubah rawak.	67 – 68		
	TP2	Mempamerkan kefahaman tentang taburan kebarangkalian.	73, 79, 81		
	TP3	Mengaplikasikan kefahaman tentang taburan kebarangkalian untuk melaksanakan tugasan mudah.	77, 80 – 81		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah rutin yang mudah.	69, 74 – 75, 77, 82 – 84		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah rutin yang kompleks.	70 – 72, 75 – 76, 78		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah bukan rutin secara kreatif.	68 – 69, 73		
Tahap Penguasaan Bab 5		**TP 1** **TP 2** **TP 3**	**TP 4**	**TP 5**	**TP 6**
6 Fungsi Trigonometri	**TP1**	Mempamerkan pengetahuan asas tentang fungsi trigonometri.	88, 89		
	TP2	Mempamerkan kefahaman tentang fungsi trigonometri.	89, 90		

BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) MENGUASAI	(✗) BELUM MENGUASAI
	TP3	Mengaplikasikan kefahaman tentang fungsi trigonometri untuk melaksanakan tugasan mudah.	91 – 94, 102 – 103		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah rutin yang mudah.	92, 94 – 95, 99 – 101, 104		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah rutin yang kompleks.	90, 95, 105 – 106		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah bukan rutin secara kreatif.	96 – 98, 106 – 107		
Tahap Penguasaan Bab 6	TP 1	TP 2	TP 3	TP 4	TP 5	TP 6
7 **Pengaturcaraan Linear**	TP1	Mempamerkan pengetahuan asas tentang model pengaturcaraan linear.	111		
	TP2	Mempamerkan kefahaman tentang model pengaturcaraan linear.	113		
	TP3	Mengaplikasikan kefahaman tentang model pengaturcaraan linear untuk melaksanakan tugasan mudah.	111 – 114		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengaturcaraan linear dalam konteks penyelesaian masalah rutin yang mudah.	114 – 115		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengaturcaraan linear dalam konteks penyelesaian masalah rutin yang kompleks.	116 – 117		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengaturcaraan linear dalam konteks penyelesaian masalah bukan rutin secara kreatif.	118 – 120		
Tahap Penguasaan Bab 7	TP 1	TP 2	TP 3	TP 4	TP 5	TP 6
8 **Kinematik Gerakan Linear**	TP1	Mempamerkan pengetahuan asas tentang sesaran, halaju dan pecutan.	123 – 124, 134		
	TP2	Mempamerkan kefahaman tentang sesaran, halaju dan pecutan.	125		
	TP3	Mengaplikasikan kefahaman tentang sesaran, halaju dan pecutan untuk melaksanakan tugasan mudah.	126 – 128, 131		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang kinematik gerakan linear dalam konteks penyelesaian masalah rutin yang mudah.	130, 132, 134 – 138		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang kinematik gerakan linear dalam konteks penyelesaian masalah rutin yang kompleks.	133, 138 – 140		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang kinematik gerakan linear dalam konteks penyelesaian masalah bukan rutin secara kreatif.	129, 141 – 143		
Tahap Penguasaan Bab 8	TP 1	TP 2	TP 3	TP 4	TP 5	TP 6

BAB 1

Sukatan Membulat
Circular Measure

KUASAI PBD FORMATIF

1.1 Radian
Radians

Buku Teks ms. 2 – 4

eP+

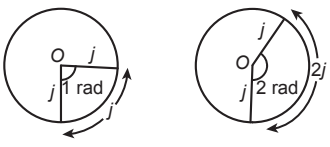
Video Tutorial

KUASAI Nota Pintas

eP+ Nota Visual

1. Sudut yang dicangkum pada pusat bulatan oleh lengkok yang sama panjang dengan jejarinya ditakrifkan sebagai 1 radian.
   1 radian is the angle subtended at the centre of the circle by the arc length which is the same length as the radius.
2. Sudut dicangkum pada pusat bulatan oleh satu bulatan ialah 2π radian yang setara dengan 360°.
   The angle subtended at the centre of a circle by the circle is 2π radians which is equivalent to 360°.
3. Hubungan antara sudut diukur dalam darjah dan radian bagi suatu sektor yang berikut:
   The relation between angle measured in degrees and radians for a sector is as follows:

$$\frac{\theta°}{360°} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} \text{ atau setaranya / } \textit{or equivalently} \frac{\theta°}{180°} = \frac{\theta \text{ rad}}{\pi \text{ rad}}$$



SP 1.1.1 Membuat perkaitan antara ukuran sudut dalam radian dengan darjah.

Video Tutorial
Radian
Radians

1. Tukarkan setiap yang berikut kepada darjah dan minit. TP 1
Convert each of the following into degrees and minutes.
Guna/*Use* $\pi = 3.142$.

Contoh

$\frac{2}{3}\pi$ rad

$\pi \text{ rad} = 180°$

$\frac{2}{3}\pi \text{ rad} = \frac{2}{3}\pi \times \frac{180°}{\pi}$

$= \frac{2}{3} \times 180°$

$= 120°$

(a) 6.8 rad

$\pi \text{ rad} = 180°$

$6.8 \text{ rad} = 6.8 \times \frac{180°}{\pi}$

$= 389° \, 34'$

(b) $\frac{7}{9}$ rad

$\pi \text{ rad} = 180°$

$\frac{7}{9} \text{ rad} = \frac{7}{9} \times \frac{180°}{\pi}$

$= \frac{7}{9} \times \frac{180°}{3.142}$

$= 44° \, 33'$

2. Tukarkan setiap yang berikut kepada radian. TP 1
Convert each of the following into radians.
Guna/*Use* $\pi = 3.142$.

Contoh

$106° \, 13'$

$180° = \pi \text{ rad}$

$106° \, 13' = 106° \, 13' \times \frac{\pi}{180°}$

$= 106° \, 13' \times \frac{3.142}{180°}$

$= 1.854 \text{ rad}$

(a) $218° \, 47'$

$180° = \pi \text{ rad}$

$218° \, 47' = 218° \, 47' \times \frac{\pi}{180°}$

$= 218° \, 47' \times \frac{3.142}{180°}$

$= 3.819 \text{ rad}$

(b) 327.5°

$180° = \pi \text{ rad}$

$327.5° = 327.5° \times \frac{\pi}{180°}$

$= 327.5° \times \frac{3.142}{180°}$

$= 5.717 \text{ rad}$

KUASAI PBD FORMATIF

1.2 Panjang Lengkok Suatu Bulatan
Arc Length of a Circle

Buku Teks ms. 5 – 11

KUASAI Nota Pintas

1. Panjang lengkok, s, suatu bulatan berkadaran dengan sudut yang tercangkum di pusat bulatan.
The arc length, s of a circle is directly proportional to the size of the angle subtended at the centre of the circle.

2. Secara am. kita boleh tulis seperti yang berikut:
In general, we can write as follows:

$$\frac{\theta°}{360°} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} = \frac{\text{panjang lengkok (}\textit{arc length}\text{)}}{\text{lilitan bulatan (}\textit{circumference}\text{)}}$$

dengan panjang lilitan $= 2\pi j$ dan jejari ialah j unit.
such that the circumference $= 2\pi j$ and the radius is j units.

3. Panjang lengkok bulatan, s, dapat ditentukan dengan menggunakan
The arc length of a circle, s, can be determined by using

$$s = j\theta$$

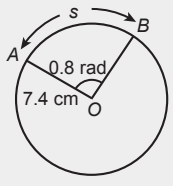
dengan j ialah jejari bulatan dan θ radian ialah sudut tercangkum oleh lengkok di pusat bulatan.
such that j is the radius of the circle and θ radian is the angle subtended by the arc at the centre of the circle.

SP 1.2.1 Menentukan (i) panjang lengkok, (ii) jejari, dan (iii) sudut tercangkum di pusat bulatan.

3. Tentukan panjang lengkok, s bagi setiap bulatan yang diberi. TP 1
Determine the arc length, s for each of the following given circles.
Guna/*Use* $\pi = 3.142$.

Video
Sukatan Membulat
Circular Measure

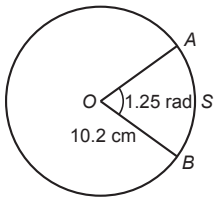
Contoh



Panjang lengkok, $s = j\theta$
Arc length

$s = 7.4 \times 0.8$
$= 5.92$ cm

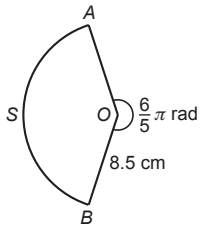
(a)



Panjang lengkok, $s = j\theta$
Arc length

$s = 10.2 \times 1.25$
$= 12.75$ cm

(b)



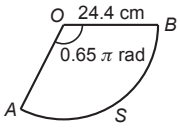
$\theta = 2\pi - \frac{6\pi}{5}$

$= \frac{4\pi}{5}$

Panjang lengkok, $s = j\theta$
Arc length

$s = 8.5 \times \frac{4\pi}{5}$
$= 21.366$ cm

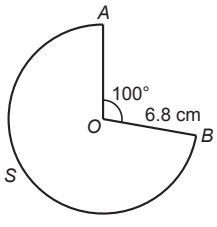
(c)



Panjang lengkok, $s = j\theta$
Arc length

$s = 0.65\pi \times 24.4$
$= 0.65 \times 3.142 \times 24.4$
$= 49.83$ cm

(d)



$\theta = 360° - 100°$
$= 260°$

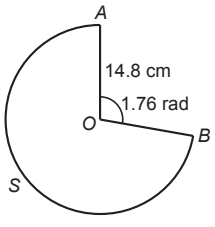
$\frac{\theta°}{360°} = \frac{s}{2\pi j}$

$\frac{\theta°}{360°} = \frac{s}{2\pi(6.8)}$

$s = \frac{260° \times 2\pi(6.8)}{360°}$

$= 30.86$ cm

(e)



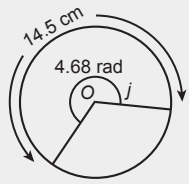
$\theta = (2\pi - 1.76)$ rad

Panjang lengkok, $s = j\theta$
Arc length

$s = 14.8 \times (2\pi - 1.76)$
$= 66.96$ cm

4. Tentukan jejari bulatan, j, diberikan panjang lengkok dan sudut bagi setiap bulatan yang berikut.
Determine the radius of the circle, j, given the arc length and the angle in each of the circles. TP 2

Contoh



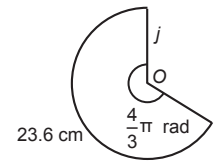
$s = j\theta$

$j = \dfrac{s}{\theta}$

$j = \dfrac{14.5}{4.68}$

$= 3.1$ cm

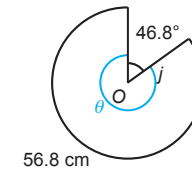
(a)



$j = \dfrac{23.6}{\frac{4}{3}\pi}$

$= 5.634$ cm

(b)



$\theta = 360° - 46.8°$

$= 313.2°$

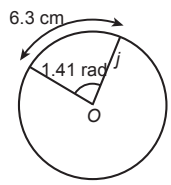
$\dfrac{\theta°}{360°} = \dfrac{s}{2\pi j}$

$\dfrac{313.2°}{360°} = \dfrac{56.8}{2\pi j}$

$j = \dfrac{56.8 \times 360°}{313.2° \times 2\pi}$

$= 10.39$ cm

(c)



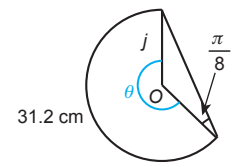
$s = j\theta$

$j = \dfrac{s}{\theta}$

$j = \dfrac{6.3}{1.41}$

$= 4.47$ cm

(d)

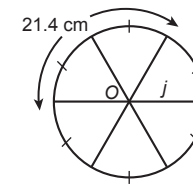


$\theta = \left[2\pi - \pi + 2\left(\dfrac{\pi}{8}\right)\right] = \dfrac{5\pi}{4}$

$j = \dfrac{31.2}{\frac{5\pi}{4}}$

$= 7.95$ cm

(e)



$\dfrac{\theta°}{360°} = \dfrac{s}{2\pi j}$

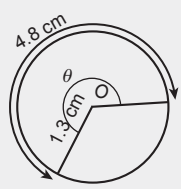
$\dfrac{120°}{360°} = \dfrac{21.4}{2\pi j}$

$j = \dfrac{21.4 \times 360°}{120° \times 2\pi}$

$= 10.22$ cm

5. Tentukan sudut tercangkum, θ dalam radian, di pusat bulatan dengan diberikan jejari bulatan dan panjang lengkok bagi setiap bulatan yang berikut. TP 3
Determine the subtended angle, θ, in radians, at the centre of the circle given that the radius and the arc length of each of the following circles.

Contoh



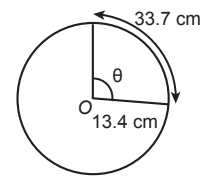
Daripada/*From* $s = j\theta$

$\theta = \dfrac{s}{j}$

$\theta = \dfrac{4.8}{1.3}$

$\theta = 3.692$ rad

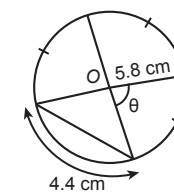
(a)



$\theta = \dfrac{33.7}{13.4}$

$\theta = 2.515$ rad

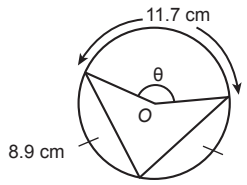
(b)



$2\pi - 3\theta = \dfrac{4.4}{5.8}$

$\theta = 1.84$ rad

(c)



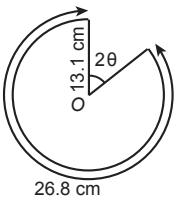
$2\pi j = 11.7 + 2(8.9)$

$j = 4.695$ cm

$\theta = \dfrac{11.7}{4.695}$

$= 2.492$ rad

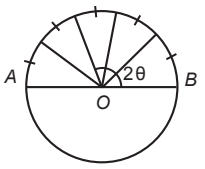
(d)



$2\pi - 2\theta = \dfrac{26.8}{13.1}$

$\theta = 2.12$ rad

(e) Diberi *AB* ialah diameter.
Given that AB is a diameter.



$2\theta = \dfrac{\pi}{5} \times 3$

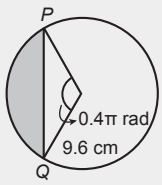
$\theta = \dfrac{3\pi}{10}$ rad

$= 0.942$ rad

SP 1.2.2 Menentukan perimeter tembereng suatu bulatan.

6. Tentukan perimeter tembereng yang berlorek bagi setiap bulatan berpusat *O* yang berikut. TP 4
Determine the perimeter of the shaded segment of each of the following circles with centre O.

Contoh



Tip Penting

Perentas *PQ* dapat diperolehi dengan petua kosinus, iaitu
$PQ = \sqrt{j^2 + j^2 - 2j^2 \text{ kos } \theta}$, dengan θ dalam darjah.
The chord PQ can be obtained by using the cosine rule, that is
$PQ = \sqrt{j^2 + j^2 - 2j^2 \cos \theta}$, *such that θ is in degrees.*

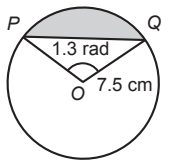
0.4π rad $= 0.4\pi \times \dfrac{180°}{\pi} = 72°$

Maka, $PQ = \sqrt{9.6^2 + 9.6^2 - 2(9.6)^2 \text{ kos } 72°}$
Hence $= 11.29$ cm

Panjang lengkok $PQ = j\theta = 9.6(0.4\pi)$
Arc length of PQ $= 12.06$ cm

Perimeter tembereng berlorek $= 12.06 + 11.29$
Perimeter of the shaded segment $= 23.35$ cm

(a)



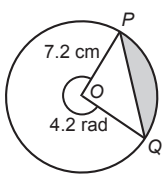
$1.3 \text{ rad} = 1.3 \times \dfrac{180°}{\pi} = 74.48°$

Maka, $PQ = \sqrt{7.5^2 + 7.5^2 - 2(7.5)^2 \text{ kos } 74.48°}$
Hence $= 9.08$ cm

Panjang lengkok $PQ = j\theta = 7.5(1.3)$
Arc length of PQ
$= 9.75$ cm

Perimeter tembereng berlorek $= 9.08 + 9.75$
Perimeter of the shaded segment $= 18.83$ cm

(b)

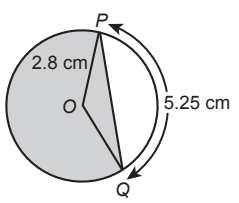


Panjang lengkok minor *PQ*
Minor arc length of PQ
$= (2\pi - 4.2) \times 7.2$
$= 15$ cm
$\angle POQ = 2\pi - 4.2 = 2.08$ rad
$= 119.36°$

Maka, $PQ = \sqrt{7.2^2 + 7.2^2 - 2(7.2)^2 \text{ kos } 119.36°}$
Hence $= 12.43$ cm

Perimeter tembereng berlorek $= 15 + 12.43$
Perimeter of the shaded segment $= 27.43$ cm

(c)



Panjang lengkok major *PQ*
Major arc length of PQ
$= 2\pi(2.8) - 5.25 = 12.34$ cm.

$\angle POQ = \dfrac{5.25}{2.8}$
$= 1.875$ rad
$= 1.875 \times \dfrac{180}{\pi}$
$= 107.43°$

Maka, $PQ = \sqrt{2.8^2 + 2.8^2 - 2(2.8)^2 \text{ kos } 107.43°}$
Hence $= 4.51$ cm

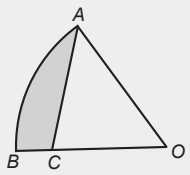
Perimeter tembereng berlorek $= 12.34 + 4.51$
Perimeter of the shaded segment $= 16.85$ cm

Cuba jawab **Praktis Sumatif 1**, *K1: S6*

SP 1.2.3 Menyelesaikan masalah yang melibatkan panjang lengkok.

7. Selesaikan masalah yang melibatkan panjang lengkok. TP 5
Solve the problems involving the arc lengths.

Contoh



Rajah menunjukkan sebuah sektor *AOB* berpusat *O*. Diberi $AC = CO = 3.4$ cm dan $\angle ACO = 1.5$ rad. Cari perimeter rantau berlorek itu.
The diagram shows a sector AOB with centre O. Given that AC = CO = 3.4 cm and ∠ACO = 1.5 rad. Find the perimeter of the shaded region.

$\angle ACO = 1.5 \text{ rad} = 85.94°$

$\angle AOB = \dfrac{\pi - 1.5}{2} = 0.82 \text{ rad}$

Jejari/*Radius*

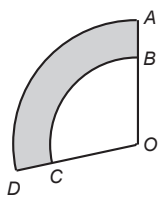
$AO = \sqrt{3.4^2 + 3.4^2 - 2(3.4)^2 \text{ kos } 85.94}$

$= 4.63 \text{ cm}$

Panjang lengkok/*Arc length* $= 4.63 \times 0.82$
$= 3.8 \text{ cm}$

Perimeter rantau berlorek $= 3.4 + 3.8 + (4.63 - 3.4)$
Perimeter of the shaded region $= 8.43 \text{ cm}$

(a)



Rajah menunjukkan dua sektor, *AOD* dan *BOC* berpusat *O*. Diberi $2OA = 3OB$, $OA = 9.6$ cm dan $\angle AOD = 2.2$ rad. Cari perimeter rantau berlorek itu.
The diagram shows two sectors, AOD and BOC with centre O. Given that 2OA = 3OB, OA = 9.6 cm and ∠AOD = 2.2 rad. Find the perimeter of the shaded region.

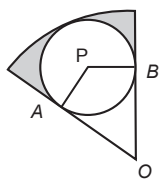
$OB = \dfrac{2}{3}(9.6) = 6.4 \text{ cm}$

Panjang lengkok $AD = 9.6(2.2) = 21.12$ cm
Arc length

Panjang lengkok $BC = 6.4(2.2) = 14.08$ cm
Arc length

Perimeter rantau berlorek $= 21.12 + 14.08 + 2(9.6 - 6.4)$
Perimeter of the shaded region $= 41.6 \text{ cm}$

(b)



Rajah menunjukkan satu sektor dengan pusat *O*. Sebuah bulatan dicangkum di dalamnya dengan pusat *P*. Diberi jejari bulatan ialah 4 cm dan $\angle APB = \dfrac{7}{8}\pi$ rad. Cari
The diagram shows a sector with centre O. A circle with centre P is inscribed in it. Given that the radius of the circle is 4 cm and ∠APB = 7/8 π rad, find

(i) jejari sektor.
the radius of the sector.

(ii) luas rantau berlorek.
the area of the shaded region.

(i) $\angle OPB = \dfrac{7}{8}\pi \times \dfrac{1}{2}$

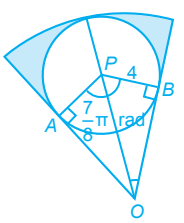
$= \dfrac{7}{16}\pi \text{ rad}$

$= 78° \, 45'$

kos / *cos* $78°45' = \dfrac{4}{OP}$

$OP = \dfrac{4}{\text{kos } 78°45'} = 20.5 \text{ cm}$

∴ Jejari sektor $= 20.5 + 4$
Radius of the sector $= 24.5 \text{ cm}$



(ii) $\angle POB = \dfrac{\pi}{2} - \dfrac{7}{16}\pi$

$= \dfrac{\pi}{16}$

$= 11.25°$

Sudut major $APB = 2\pi - \dfrac{7}{8}\pi$
Major angle APB $= \dfrac{9\pi}{8}$

∴ Luas rantau berlorek / *Area of the shaded region*

$= \dfrac{22.5°}{360°} \times \pi(24.5)^2 - 2\left[\dfrac{1}{2}(4)(20.5) \sin 78°45'\right] - \dfrac{1}{2}(4)^2\left(\dfrac{9\pi}{8}\right)$

$= 9.17 \text{ cm}^2$

Cuba jawab **Praktis Sumatif 1**, *K1: S5*

KUASAI PBD FORMATIF

1.3 Luas Sektor Suatu Bulatan
Area of Sector of a Circle

Buku Teks ms. 12 – 19

KUASAI Nota Pintas

Video Tutorial
Luas Sektor Suatu Bulatan
Sector Area of a Circle

1. Hubungan secara am antara sudut, panjang lengkok dan luas sektor adalah seperti berikut,
The relationship between angles, arc length and the area are as follows

$$\frac{\theta°}{360°} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} = \frac{\text{panjang lengkok / } \textit{arc length, s}}{2\pi j} = \frac{\text{luas sektor / } \textit{area of sector}}{\text{luas bulatan / } \textit{area of circle}}$$

2. Luas sektor bulatan, L, dapat ditentukan dengan menggunakan
The area of a sector of the circle, L, can be determined by using

$$L = \frac{1}{2}j^2\theta$$

dengan j ialah jejari bulatan dan θ radian ialah sudut tercangkum oleh sektor di pusat bulatan.
where j is the radius of the circle and θ radian is the angle subtended by the sector at the centre of the circle.

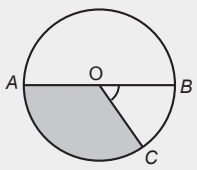
Info
Cara mencari rumus panjang perentas, luas sektor dan luas tembereng.
Steps to find the length of chord, the area of sector and the area of segment.

SP 1.3.1 Menentukan (i) luas sektor, (ii) jejari, dan (iii) sudut tercangkum di pusat bulatan.

Video Tutorial

8. Tentukan luas sektor berlorek bagi setiap bulatan yang berikut. Beri jawapan betul kepada dua tempat perpuluhan.
Determine the area of the shaded sector for each of the following circles. Give your answer correct to two decimal places. TP 3

Contoh



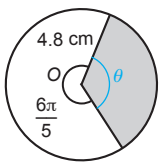
Diberi *AOB* ialah diameter dengan panjangnya ialah 7.4 cm, dan $\angle AOC = 2\angle BOC$.
Given AOB is the diameter of length 7.4 cm and $\angle AOC = 2\angle BOC$.

$$\angle AOC = \frac{2\pi}{3} \text{ rad}$$

$$L = \frac{1}{2}j^2\theta$$

$$= \frac{1}{2}(3.7)^2\left(\frac{2\pi}{3}\right) = 14.34 \text{ cm}^2$$

(a)



$$\theta = \left(2\pi - \frac{6\pi}{5}\right) \text{ rad}$$

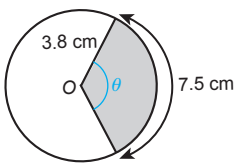
$$= \frac{4\pi}{5} \text{ rad}$$

$$L = \frac{1}{2}j^2\theta$$

$$= \frac{1}{2}(4.8)^2\left(\frac{4\pi}{5}\right)$$

$$= 28.95 \text{ cm}^2$$

(b)



$$\theta = \frac{7.5}{3.8}$$

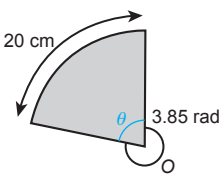
$$= 1.97 \text{ rad}$$

$$L = \frac{1}{2}j^2\theta$$

$$= \frac{1}{2}(3.8)^2(1.97)$$

$$= 14.22 \text{ cm}^2$$

(c)



$$\theta = (2\pi - 3.85)$$

$$= 2.43 \text{ rad}$$

$$\text{Jejari / } \textit{Radius} = \frac{20}{2.43} = 8.23 \text{ cm}$$

$$L = \frac{1}{2}j^2\theta$$

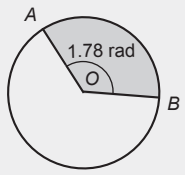
$$= \frac{1}{2}(8.23)^2(2.43)$$

$$= 82.30 \text{ cm}^2$$

9. Tentukan jejari bagi sektor bulatan berlorek yang berikut. Beri jawapan betul kepada dua tempat perpuluhan.
Determine the radius of the following shaded sectors of circles. Give your answer correct to two decimal places. TP 4

Contoh

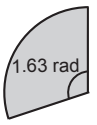
Diberi luas sektor yang berlorek ialah 23.8 cm²
Given the area of the shaded sector is 23.8 cm².



Tip Penting
Guna $\frac{\text{luas sektor}}{\text{luas bulatan}} = \frac{\theta}{2\pi}$
Use $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\theta}{2\pi}$

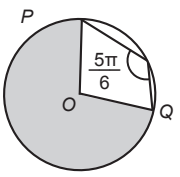
$$\frac{23.8}{\pi j^2} = \frac{1.78}{2\pi}$$
$$j^2 = 23.8 \times \frac{2}{1.78}$$
$$j = 5.17 \text{ cm}$$

(a) Diberi luas sektor ialah 49 cm²
Given the area of the sector is 49 cm².



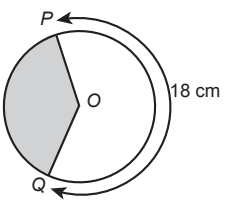
$$\frac{49}{\pi j^2} = \frac{1.63}{2\pi}$$
$$j^2 = \frac{49 \times 2}{1.63}$$
$$j = 7.75 \text{ cm}$$

(b) Diberi luas sektor berlorek ialah 7.5 cm².
Given the area of the shaded sector is 7.5 cm².



$$\frac{7.5}{\pi j^2} = \frac{2\left(\frac{5\pi}{6}\right)}{2\pi}$$
$$\frac{7.5}{\pi j^2} = \frac{\frac{5\pi}{3}}{2\pi}$$
$$j^2 = \frac{7.5 \times 2 \times 3}{5\pi}$$
$$j = 1.69 \text{ cm}$$

(c)



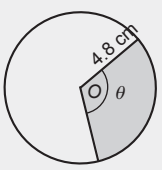
Diberi luas sektor berlorek ialah 30 cm² dan panjang lengkok major *PQ* ialah 18 cm.
Given the area of the shaded sector is 30 cm² *and the major arc length is* 18 cm.

Tip Penting
Guna/ *Use*
$\frac{\text{panjang lengkok}}{\text{lilitan bulatan}} = \frac{\text{luas sektor}}{\text{luas bulatan}}$
$\frac{\text{arc length}}{\text{circum ference}} = \frac{\text{area of sector}}{\text{area of circle}}$

$$\frac{18}{2\pi j} = \frac{\pi j^2 - 30}{\pi j^2}$$
$$j = \frac{2(\pi j^2 - 30)}{18}$$
$$\pi j^2 - 30 = 9j$$
$$\pi j^2 - 9j - 30 = 0$$
$$j = \frac{9 \pm \sqrt{(-9)^2 - 4\pi(-30)}}{2\pi}$$
$$j = 4.84 \text{ cm}$$

10. Tentukan sudut tercangkum, θ dalam radian, di pusat bulatan bagi setiap yang berikut. Beri jawapan betul kepada dua tempat perpuluhan jika perlu. TP 4
Determine the subtended angle, θ in radians, at the centre of the circle for each of the following. Give your answer correct to two decimal places where necessary.

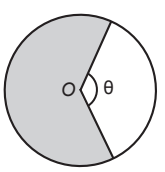
Contoh



Diberi luas sektor berlorek ialah 23 cm²
Given the area of the shaded sector is 23 cm².

$$\frac{23}{\pi(4.8)^2} = \frac{\theta}{2\pi}$$
$$\theta = \frac{23 \times 2}{(4.8)^2}$$
$$= 1.99 \text{ rad}$$

(a)



Diberi luas sektor berlorek ialah 15.7 cm² dan berjejari 4.2 cm.
Given the area of the shaded sector is 15.7 cm² *and with radius* 4.2 cm.

$$\frac{15.7}{\pi(4.2)^2} = \frac{2\pi - \theta}{2\pi}$$
$$\theta = 2\pi - \frac{15.7 \times 2}{(4.2)^2}$$
$$= 4.50 \text{ rad}$$

(b) Diberi luas sektor ialah 40 cm² dan perimeternya ialah 28 cm. Cari nilai yang mungkin bagi jejari sektor dan sudut sepadan tercangkum.
Given the area of the sector is 40 cm² and the perimeter is 28 cm, Find the possible values of the radius of the sector and the corresponding subtended angle.

$j + j + j\theta = 28$

$2j + j\theta = 28$ dan $j\theta = 28 - 2j$ ..... ①

$\frac{1}{2}j^2\theta = 40$

$j^2\theta = 80$ ..... ②

$j(28 - 2j) = 80$

$j^2 - 14j + 40 = 0$

$(j - 4)(j - 10) = 0$

$j = 4$ atau $j = 10$ cm

$\theta = 5$ rad atau 0.8

(c) Diberi luas sektor ialah 6.25 cm² dan perimeternya ialah 12.5 cm. Cari jejari dan sudut sepadan tercangkum.
Given the area of the sector is 6.25 cm² and the perimeter is 12.5 cm. Find the radius and the corresponding subtended angle.

$j + j + j\theta = 12.5$

$2j + j\theta = 12.5$ dan $j\theta = 12.5 - 2j$ ..... ①

$\frac{1}{2}j^2\theta = 6.25$

$j^2\theta = 12.5$ ..... ②

$j(12.5 - 2j) = 12.5$

$2j^2 - 12.5j + 12.5 = 0$

$4j^2 - 25j + 25 = 0$

$(4j - 5)(j - 5) = 0$

$j = \frac{5}{4}$ atau $j = 5$ cm

$\theta = 8$ rad (diabaikan) atau 0.5 rad

Ukuran sudut pada pusat tidak boleh melebihi $360° = 2\pi$ rad = 6.28 rad

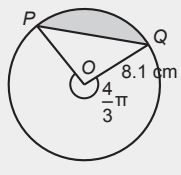
The measurement of a central angle cannot be greater than $360° = 2\pi$ rad = 6.28 rad

Maka, $j = 5$ cm dan $\theta = 0.5$ rad

SP 1.3.2 Menentukan luas tembereng suatu bulatan

11. Tentukan luas tembereng yang berlorek bagi setiap bulatan yang berikut. Beri jawapan betul kepada dua tempat perpuluhan. TP 5
Determine the area of the shaded segment for each of the following circles. Give your answer correct to two decimal places.

Contoh



Luas sektor $= \frac{1}{2}j^2\left(2\pi - \frac{4\pi}{3}\right)$
Area of sector

$= \frac{1}{2}(8.1)^2\left(\frac{2\pi}{3}\right)$

$= 68.71$ cm²

$\frac{2\pi}{3}$ rad $= 120°$

Luas segi tiga $= \frac{1}{2}(8.1)^2 \sin 120°$
Area of the triangle

$= 28.41$ cm²

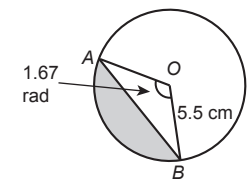
Maka, luas tembereng berlorek $= 68.71 - 28.41$
Hence, the area of shaded segment

$= 40.30$ cm²

Tip Penting

Luas tembereng berlorek/ *Area of shaded segment*
= luas sektor *POQ* – luas segi tiga *POQ*
area of sector POQ – area of triangle POQ

(a)



Luas sektor $= \frac{1}{2}j^2\theta$
Area of sector

$= \frac{1}{2}(5.5)^2(1.67)$

$= 25.26$ cm²

Luas segi tiga $= \frac{1}{2}(5.5)^2 \sin 95.68°$
Area of triangle

$= 15.05$ cm²

Maka, luas tembereng berlorek $= 25.26 - 15.05$
Hence, the area of the shaded segment

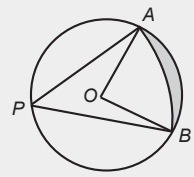
$= 10.21$ cm²

Cuba Jawab Praktis Sumatif 1, *K1: S2*

SP 1.3.3 Menyelesaikan masalah yang melibatkan luas sektor.

12. Selesaikan yang berikut. TP 5
Solve the following.

Contoh



Rajah menunjukkan sebuah bulatan dengan pusat O dan jejari 8 cm dan $\angle AOB = \frac{2\pi}{3}$ rad. Jika P terletak pada lilitan bulatan ialah pusat sektor APB, cari

The diagram shows a circle with centre O and radius 8 cm and $\angle AOB = \frac{2\pi}{3}$ rad. If P lies on the circumference of the circle is the centre of sector APB, find

(i) $\angle APB$ dalam radian.
$\angle APB$ in radians.

(ii) luas rantau berlorek.
the area of the shaded region.

(i) $\angle APB = \frac{1}{2}\left(\frac{2\pi}{3}\right) = \frac{\pi}{3}$ rad

(ii) Luas rantau berlorek/*Area of the shaded region*

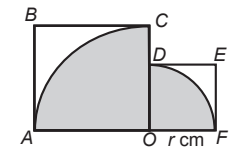
$$= \left[\frac{1}{2}(8^2)\left(\frac{2\pi}{3}\right) - \frac{1}{2}(8^2)\sin 120°\right] - \left[\frac{1}{2}(PA^2)\left(\frac{\pi}{3}\right) - \frac{1}{2}(PA^2)\sin 60°\right]$$

Tetapi/*But* $AP = 2(8 \text{ kos } 30°) = 13.86$ cm
Maka, luas rantau berlorek
Hence, the area of the shaded region

$$= \left[\frac{1}{2}(8^2)\left(\frac{2\pi}{3}\right) - \frac{1}{2}(8^2)\sin 120°\right] - \left[\frac{1}{2}(13.86^2)\left(\frac{\pi}{3}\right) - \frac{1}{2}(13.86^2)\sin 60^0\right]$$

$= 21.91 \text{ cm}^2$

(a)



Rajah menunjukkan dua segi empat sama. Panjang AO ialah 2 cm lebih panjang daripada OF. Diberi bahawa panjang lengkung AC ialah dua kali panjang lengkung DF, cari

The diagram shows two squares. The length AO is 2 cm longer than the length OF. Given that the arc length AC is twice as long as the arc length DF, find

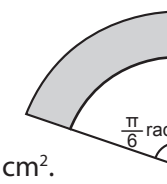
(i) nilai r./*the value of r.*

(ii) luas rantau berlorek dalam sebutan π.
the area of the shaded region in terms of π.

(i) $AC = 2DF$
$(r + 2) = 2r$
$r = 2$ cm

(ii) Luas rantau berlorek
Area of the shaded region
$= \frac{1}{4}\pi(4^2) + \frac{1}{4}\pi(2)^2$
$= 5\pi \text{ cm}^2$

(b) Rajah menunjukkan dua sektor berpusat O. Diberi bahawa jejari sektor besar ialah 3 cm lebih panjang daripada jejari sektor kecil dan luas berlorek ialah $\frac{13\pi}{4}$ cm^2.



Cari nilai jejari sektor kecil.

The diagram shows two sectors with centre O. Given that radius of the larger sector is 3 cm longer than the smaller sector and the area of the shaded region is $\frac{13\pi}{4}$ cm^2. Find the radius of the smaller sector.

Katakan jejari sektor kecil ialah r cm.
Let the radius of the smaller sector be r cm.

$$\frac{1}{2}(r+3)^2\left(\frac{\pi}{6}\right) - \frac{1}{2}(r)^2\left(\frac{\pi}{6}\right) = \frac{13\pi}{4}$$

$6r + 9 = 39$

$r = 5$ cm

Cuba jawab **Praktis Sumatif 1**, *K2: S2*

KUASAI PBD FORMATIF

1.4 Aplikasi Sukatan Membulat
Application of Circular Measures

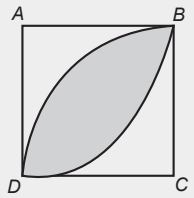
Buku Teks ms. 20 – 27

SP 1.4.1 Menyelesaikan masalah yang melibatkan sukatan membulat.

Video Tutorial
Aplikasi Sukatan Membulat
Application of Circular Measures

13. Selesaikan masalah yang berikut. TP 5
Solve the following problems.

Contoh



Rajah menunjukkan sekeping jubin bersegi empat sama dengan sisi 20 cm. *A* dan *C* masing-masing ialah pusat bagi sektor *BAD* dan *BCD*. Cari
The diagram shows a piece of square tile with side of 20 cm. *A and C are the centres of the sectors BAD and BCD respectively. Find*

(i) luas rantau berlorek.
the area of the shaded region.

(ii) jumlah luas, dalam m², diliputi oleh rantau berlorek jika dimensi bilik ialah 6 m × 5 m
the total area, in m² *covered by the shaded region if the dimension of the room is* 6 m × 5 m.

Tip Penting
Luas rantau berlorek/*Area of the shaded region*
= 2(luas sektor *BAD* – luas segi tiga *BAD*)
2(area of sector BAD – area of triangle BAD)

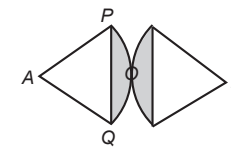
(i) Luas rantau berlorek
Area of the shaded region

$$= 2\left[\frac{1}{2}(20^2)\left(\frac{\pi}{2}\right) - \frac{1}{2}(20^2)\sin 90°\right]$$

$= 228.32 \text{ cm}^2$

(ii) Jumlah jubin diperlukan = 30 × 25 = 750
Total tiles needed
Jumlah luas diliput oleh rantau berlorek
Total area covered by shaded region
$= 228.32 \times 750 = 171\,240 \text{ cm}^2$
$= 17.124 \text{ m}^2$

(a)

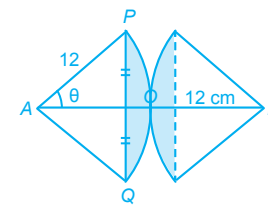


Rajah menunjukkan satu corak yang dibina daripada dua sektor serupa menyentuh satu sama lain di *O* dan dengan pusat masing-masing *A* dan *B*. Diberi panjang perentas *PQ* ialah 16 cm dan *AB* = 24 cm. Cari
The diagram shows a pattern made up of two similar sectors with centres A and B respectively and touch each other at O. Given that the chord PQ is 16 cm *and AB* = 24 cm. *Find*

(i) ∠*PAQ dalam radian.*
∠*PAQ in radians.*

(ii) perimeter rantau berlorek.
the perimter of the shaded region.

(iii) luas rantau berlorek.
the area of the shaded region.



(i) $\sin\theta = \frac{8}{12}$
$= 41.81° = 0.73 \text{ rad}$

$\angle PAQ = 1.46 \text{ rad}$

(ii) Perimeter rantau berlorek
Perimeter of the shaded region
$= 2[16 + 12(1.46)]$
$= 67.04 \text{ cm}$

(iii) Luas rantau berlorek
Area of shaded region

$$= 2\left[\frac{1}{2}(12)^2(1.46) - \frac{1}{2}(12)^2 \sin 83.62°\right]$$

$= 67.13 \text{ cm}^2$

Cuba jawab **Praktis Sumatif 1**, *K2: S1*

Video Tutorial

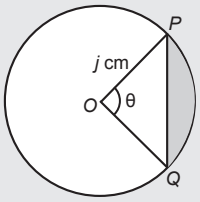
14. Lakukan aktiviti yang berikut. TP 6
Perform the following activities.

AKTIVITI **PAK-21**

Fikir-Pasang-Kongsi

(a) Lakukan kerja dalam kumpulan.
Work in groups.

(b) Guru mengemukakan soalan kepada ahli pasukan.
The teacher gives the following question to the group members.



Tentukan perimeter dan luas bagi tembereng yang diberi dengan menggunakan sekurang-kurangnya dua kaedah yang berlainan.
Determine the perimeter and area of the given segment by using at least two different methods.

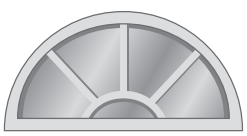
(c) Setiap ahli diberi masa untuk berfikir, dan berbincang dengan ahli pasukan yang lain.
Each member is given some time to think and then share with other members of the group.

(d) Semua pasukan pula diminta berkongsi jawapan antara satu sama lain.
All the groups share their answers with one another.

15. Lakukan projek STEM di bawah. TP 6
Cary out the STEM project below.

Projek STEM

Project-based learning

Objektif aktiviti: ***Activity objective:***	Mengaplikasi sukatan membulat dalam kehidupan seharian. *Apply the circular measure into daily lives.*
Pernyataan masalah: ***Problem statement:***	Cari luas setiap jenis struktur kaca untuk membentuk hiasan pintu . *Find the area of each type of the glass structures that makes up the design of the door.*
Pencarian fakta: ***Fact finding:***	Cari jenis fungsi ukuran membulat yang sesuai untuk menyelesaikan masalah ini. *Find the suitable circular measurement of function that is suitable to solve this problem.*
Konsep yang diaplikasikan: ***Concept applied:***	Ukuran membulat yang berkait dengan mencari luas. *Circular measures that are related to find areas.*
Bahan yang diperlukan: ***Materials needed:***	 

Pelan Tindakan/ ***Action plan:***

(a) Bahagikan kelas kepada beberapa kumpulan yang terdiri daripada 4 orang
Divide the class into groups of 4 students.

(b) Pilih salah satu daripada dua gambar rajah di atas. Diberi bahawa setiap gambar terdiri daripada semibulatan besar berjejari 1 m sebenar. Salin gambar itu dan menurut corak sebenar bagi setiap bahagian corak yang berlainan, cari luas setiap jenis struktur yang berlainan yang membina corak itu.
Choose one of the above two diagrams. Given that the radius of the big semicircle is 1 m. Copy the diagram and follow its original proportions for different part of the design, find the area of each type of glass structure that made up the design.

(c) Bentangkan hasil dapatan dalam kelas.
Present the findings in class.

Penyelesaian: ***Solutions:***	Salin rajah mengikut sukatannya untuk setiap corak. Selepas itu, gunakan rumus yang sesuai untuk mencari luas setiap jenis corak struktur kaca untuk membina rekaan itu. Unit radian lebih berguna dalam pengiraan seni bina. *Copy the diagram according to its proportions for each pattern. Then, use appropriate formula to find the area of each pattern of glass structure that makes up the design. Radians are more useful in architecture design calculations.*

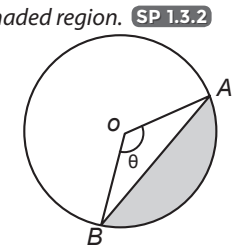
STEM

AKTIVITI PAK-21

SP 1.4.1
TAHAP PENGUASAAN 1 2 3 4 5 6

KERTAS 1

1. Rajah menunjukkan panjang jejari bulatan ialah 7 cm dan θ ialah 1.8 rad, cari luas rantau berlorek.
The diagram shows the radius of circle is 7 cm and θ is 1.8 rad, find the area of shaded region. SP 1.3.2



[3 *markah* / 3 *marks*]

Jawapan / *Answer*:

Luas rantau berlorek = luas sektor *AOB* – luas segi tiga *AOB*

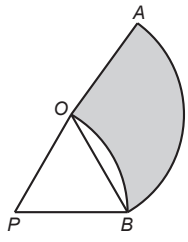
Area of shaded region = area of sector AOB – area of triangle AOB

$= \frac{1}{2}j^2\theta - \frac{1}{2}j^2 \sin\theta$

$= \frac{1}{2}(7)^2(1.8) - \frac{1}{2}(7)^2 \sin 103.13°$

$= 20.24 \text{ cm}^2$

2. Rajah menunjukkan dua sektor *AOB*, dengan pusat *O*, dan *OPB* dengan pusat *P*.
The diagram shows two sectors AOB, with centre O, and OPB with centre P. SP1.2.1

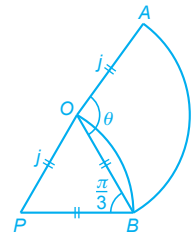


Diberi jejari $PO = j$ cm, $\angle PBO = \frac{\pi}{3}$ rad, panjang lengkok *AB* ialah dua kali panjang jejari *OA* dan perimeter rantau berlorek ialah $24 + \frac{8\pi}{3}$ cm. Cari nilai *j*.

Given that PO = j cm, $\angle PBO = \frac{\pi}{3}$ rad, the arc length AB is twice the length of the radius OA and the perimeter of the shaded region is $24 + \frac{8\pi}{3}$ cm. Find the value of j.

[5 *markah* / 5 *marks*]

Jawapan / *Answer*:



$j\theta = AB = 2j$

$\therefore \theta = 2$ rad

Perimeter $= j\left(\frac{\pi}{3}\right) + j + 2j$

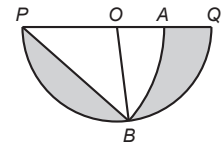
$= 24 + \frac{8\pi}{3}$

$3j + \frac{\pi}{3}j = 24 + \frac{8\pi}{3}$

$j\left(3 + \frac{\pi}{3}\right) = 8\left[3 + \frac{\pi}{3}\right]$

$\therefore j = 8$

3. Rajah menunjukkan sebuah semibulatan berpusat *O* dan sebuah sektor berpusat *P*.
The diagram shows a semicircle with centre O and a sector with centre P.



Diberi bahawa *A* ialah titik tengah bagi *OQ* dan jika perimeter rantau berlorek ialah $(50 + 20\pi)$ cm, cari SP1.3.1
Given that A is the midpoint of OQ, and if the perimeter of the shaded region is $(50 + 20\pi)$ cm, find

(a) jejari *PA*
radius PA

[3 *markah* / 3 *marks*]

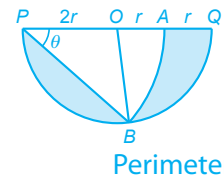
(b) $\angle APB$ [2 *markah* / 2 *marks*]

(c) luas rantau berlorek.
area of the shaded region.

[3 *markah* / 3 *marks*]

Jawapan / *Answer*:

(a)



Perimeter $= 50 + 20\pi$

$\pi(2r) + 3r + 3r\theta + r = 50 + 20\pi$

$2\pi r + 4r + 3r\theta = 50 + 20\pi$

Bandingkan

$2\pi r = 20\pi$

$r = 10$ cm

$PA = 3r = 30$ cm

(b) $4r + 3r\theta = 50$

$40 + 30\theta = 50$

$30\theta = 10$

$\theta = \frac{1}{3}$ rad

(c) Luas berlorek = luas semibulatan – luas sektor APB

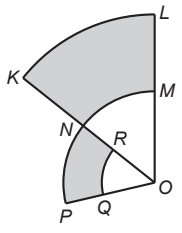
Shaded area = Area of semicircle – Area of sector APB

$= \frac{1}{2}\pi(20)^2 - \frac{1}{2}(30)^2\left(\frac{1}{3}\right)$

$= 478.4\text{ cm}^2$

4. Rajah menunjukkan dua sektor, *KOL* dan *PON* berpusat *O*.

The diagram shows two sectors, KOL and PON with centre O.



Diberi bahawa *M* dan *Q* masing-masing ialah titik tengah bagi *OL* dan *OP*. Jika perimeter *KLMN* ialah dua kali perimeter *PQRN*, SP 1.3.2

Given that M and Q are midpoints for OL and OP respectively. If the perimeter of KLMN is twice the perimeter of PQRN,

(a) ungkapkan sudut α bagi $\angle KOL$ dalam sebutan β bagi $\angle PON$.

express the angle α for $\angle KOL$ in terms of β for $\angle PON$.

[4 *markah* / 4 *marks*]

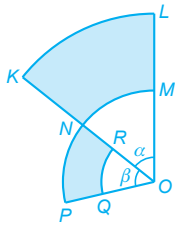
(b) cari nisbah luas *PQRN* kepada luas *KNML*.

find the ratio of the area PQRN to the area KNML.

[4 *markah* / 4 *marks*]

Jawapan / *Answer*:

(a)



Perimeter *KLMN* = 2 perimeter *PQRN*

Katakan $OQ = j$

Maka,

$2j + 4j\alpha + 2j + 2j\alpha = 2[j + 2j\beta + j + j\beta]$

$4j + 6j\alpha = 2[2j + 3j\beta]$

$= 4j + 6j\beta$

$\therefore \alpha = \beta$

(b) Luas/*Area PQRN* $= \frac{1}{2}(2j)^2\alpha - \frac{1}{2}j^2\alpha$

$= \frac{3}{2}j^2\alpha$

Luas/*Area KNML* $= \frac{1}{2}(4j)^2\alpha - \frac{1}{2}(2j)^2\alpha$

$= 6j^2\alpha$

Nisbah/*Ratio PQRN : KNML* $= \frac{3}{2}j^2\alpha : 6j^2\alpha$

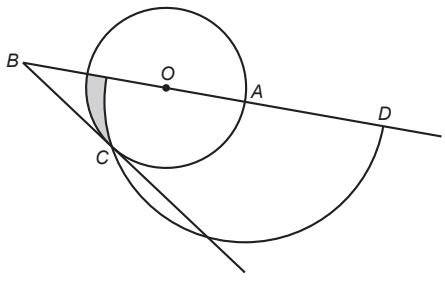
$= 1 : 9$

KERTAS 2

1. Rajah menunjukkan sebuah bulatan dengan pusat *O* dan sebuah semibulatan dengan pusat *A*. *BOAD* ialah satu garis lurus dan *BC* ialah satu tangen kepada bulatan pada titik *C*. Semibulatan dan bulatan tersebut bersilang pada titik *C*.

The diagram shows a circle with centre O and a semicircle with centre A. BOAD is a straight line and BC is a tangent to the circle at C. The semicircle and the circle intersect at C.

KBAT Menganalisis



Diberi bahawa panjang lengkok minor *AC* ialah 5.65 cm dan $\angle COA = 123°$

Given that minor arc length AC is 5.65 cm and $\angle COA = 123°$,

(a) cari nilai *h*, betul kepada dua tempat perpuluhan jika $BA - BC = h$ cm SP 1.2.3

find the value of h, correct to 2 decimal places if $BA - BC = h$ cm

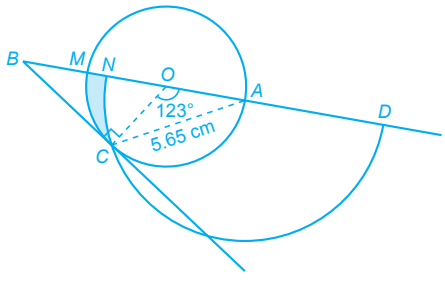
[3 *markah* / 3 *marks*]

(b) tunjukkan bahawa luas rantau berlorek , betul kepada dua tempat perpuluhan ialah 1.04 cm^2 SP 1.3.2

show that the area of the shaded region, correct to two decimal places is 1.04 cm^2

[5 *markah* / 5 *marks*]

Jawapan / *Answer*:



KBAT

(a) $123° = \frac{123°}{360°} \times 2\pi = 2.147$ rad,

jejari/*radius* $OC = \frac{5.65}{2.147} = 2.63$ cm

$\tan(180° - 123°) = \frac{BC}{2.63}$

$\therefore BC = 4.05$ cm

kos/*cos* $57° = \frac{OC}{BO}$

$BO = \frac{2.63}{\text{kos}/\cos 57°} = 4.83$ cm

$h = BA - BC = (4.83 + 2.63) - 4.05 = 3.41$ cm

(b) Luas semibulatan $OMCA = \frac{1}{2}\pi(2.63)^2$
Area of semicircle OMCA $= 10.87$ cm^2

Luas sektor $NAC = \frac{28.5°}{360°} \times \pi(AC)^2$

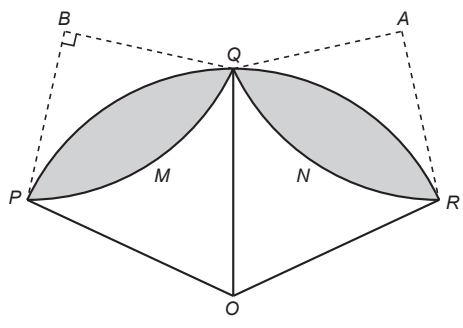
Tetapi $AC^2 = 2.63^2 + 2.63^2 - 2(2.63)^2 \text{kos}/\cos 123°$
$= 21.37$

$\therefore$ Luas sektor $NAC = \frac{28.5°}{360°} \times \pi(21.37)$
$\therefore$ *Area of sector NAC* $= 5.31$ cm^2

Luas tembereng $AC = \frac{123°}{360°} \times \pi(2.63^2) - \frac{1}{2}(2.63)^2 \sin 123°$
Area of segment AC $= 4.52$ cm^2

Luas rantau berlorek $= 10.87 - 5.31 - 4.52$
Area of the shaded region $= 1.04$ cm^2

2. Rajah menunjukkan suatu mural pada dinding bilik yang dilukis oleh John. Mural itu terdiri daripada dua sektor, *POQ* dan *QOR* dengan pusat *O* dan jejari 70 cm.
The diagram shows a mural on the wall in a room that is drawn by John. The mural is made up of two sectors, POQ and QOR with centre O and radius 70 cm.



Panjang lengkok *PQ* = panjang lengkok *QR* = 80 cm. Panjang lengkok *PMQ* = panjang lengkok *QNR* = 70 cm, dengan keadaan *A* dan *B* masing-masing ialah pusat sektor dan $\angle PBQ = 90°$. Cari
The arc length PQ = the arc length QR = 80 cm. The arc length of PMQ = the arc length of QNR = 70 cm, such that A and B are the centres respectively and ∠PBQ = 90°. Find

(a) sudut *POQ*.
the angle POQ.

[1 *markah* / 1 *mark*]

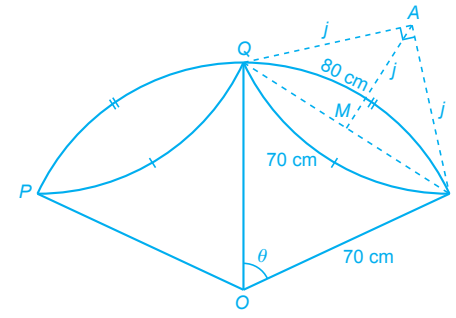
(b) panjang perentas *PQ*.
the length of the chord PQ.

[2 *markah* / 2 *marks*]

(c) John ingin mengecat rantau berlorek itu dengan biru dan selainnya dengan putih. Cari luas yang dicat putih.
John wants to paint the shaded region blue and the rest white. Find the area painted in white.

[4 *markah* / 4 *marks*]

Jawapan / *Answer*:



(a) $80 = 70\theta$

$\theta = \frac{8}{7}$rad, $= 65.48°$

(b) $PQ^2 = 70^2 + 70^2 - 2(70)^2 \text{kos}/\cos 65.48°$

$PQ = 75.72$ cm

$70 = j\left(\frac{\pi}{2}\right)$

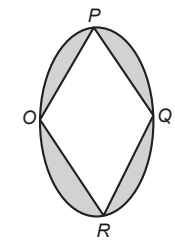
$j = \frac{70 \times 2}{\pi} = 44.56$ cm

(c) Luas rantau berlorek $= \left[\frac{1}{2}(70^2)\left(\frac{8}{7}\right) - \frac{1}{2}(70^2)\sin 65.48°\right] + \left[\frac{1}{2}(44.56)^2\left(\frac{\pi}{2}\right) - \frac{1}{2}(44.56)^2 \sin 90°\right]$
Area of shaded region
$= 570.95 + 566.68$
$= 1137.63$ cm^2

Luas putih $= 2\left[\frac{1}{2}(70)^2\left(\frac{8}{7}\right) - 1137.63\right]$
Area in white $= 3324.74$ cm^2

3. Rajah menunjukkan satu cermin *OPQR* dan bahagian berlorek ialah kayu hiasan yang terdiri daripada dua sektor *OPQR* yang berpusat *O* dan *QPOR* yang berpusat Q dengan jejari sama, *j*.
The diagram shows a mirror OPQR and the shaded area of a decorative wood frame which is made up of two sectors, OPQR with centre O and QPOR with centre Q and of the same radius, j.

SP 1.4.1 KBAT Menganalisis



KBAT

(a) Tunjukkan bahawa $\angle POR = \frac{2\pi}{3}$ radian.
Show that $\angle POR = \frac{2\pi}{3}$ *radian.*
[5 *markah* / 5 *marks*]

(b) Cari jumlah perimeter kayu hiasan jika jejari sektor ialah 0.5 m.
Find the total perimeter of the decorative wood if the radius of the sector is 0.5 m.
[3 *markah* / 3 *marks*]

TIP Menjawab
Perhatikan bahawa $OP = OQ = OR$.
Maka, keempat-empat tembereng ialah bersaiz sama.
It is observed that $OP = OQ = OR$.
Hence, the four segments are equal in size.

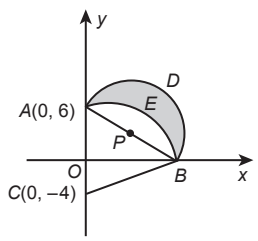
Jawapan / *Answer*:

(a) Oleh sebab/ *Since*
$OP = PQ = OQ$
Maka/ *Hence* $\angle POQ$
$= 60°$
$= \frac{\pi}{3}$
$\angle POR = \frac{2\pi}{3}$ rad.

(b) panjang lengkok PQ
arc length of PQ
$= 0.5\left(\frac{\pi}{3}\right) = \frac{\pi}{6}$ cm

Jumlah perimeter
Total perimeter
$= 4(0.5) + 4\left(\frac{\pi}{6}\right) = 4.09$ m

4. Rajah menunjukkan satu semibulatan $PADB$ berpusat P dan sektor CAB berpusat C dilukis pada satah Cartes.
The diagram shows a semicircle PADB with centre P and a sector CAB with centre C drawn on a Cartesian plane.



Diberi $CP = 5\sqrt{3}$ unit, cari SP 1.4.1
Given that $CP = 5\sqrt{3}$ unit, *find*

(a) diameter semibulatan itu.
the diameter of the semicircle.
[2 *markah* / 2 *marks*]

(b) sudut ACB, dalam radian.
the angle ACB, in radians.
[2 *markah* / 2 *marks*]

(c) perimeter rantau berlorek.
the perimeter of the shaded region.
[3 *markah* / 3 *marks*]

(d) luas rantau berlorek.
the area of the shaded region.
[3 *markah* / 3 *marks*]

(a) $CP = 5\sqrt{3}$
$CA = 10 = CB$
$\therefore AP = \sqrt{10^2 - 25 \times 3}$
$= 5$ units

Diameter $AB = 10$ units

(b) $\sin \angle PCB = \frac{5}{10} = \frac{1}{2}$
$\angle PCB = 30°$
$\angle ACB = 60° = \frac{\pi}{3}$ rad

(c) Panjang lengkok $ADB = \pi(5)$ unit
Arc length ADB

Panjang lengkok $AEB = \frac{\pi}{3}(10)$ unit
Arc length AEB

Perimeter rantau berlorek $= \frac{10}{3}\pi + 5\pi$
Perimeter of the shaded region
$= 26.18$ unit

(d) Luas rantau berlorek
Area of the shaded region
= luas semibulatan – luas tembereng
area of semicircle – area of segment
$= \frac{1}{2}\pi(5)^2 - \left[\frac{1}{2}(10)^2\frac{\pi}{3} - \frac{1}{2}(10^2)\sin 60°\right]$
$= 30.21$ unit^2

Aplikasi KBAT

POT
Enrolment key
KUA$aim+5

BAB 3

Pengamiran
Integration

KUASAI PBD FORMATIF

3.1 Pengamiran Sebagai Songsangan Pembezaan
Integration as the Inverse of Differentiation

Buku Teks ms. 82 – 84

KUASAI Nota Pintas

1. Songsangan bagi pembezaan dikenali sebagai pengamiran.
   An inverse to differentiation is known as integration.
2. Notasi bagi pengamiran ialah
   The notation for integration is

   Notasi pengamiran *Integration notation* → $\int f(x)\,dx$ ← Terhadap x/ *With respect to x*

   Fungsi/ *Function*

3. Pembezaan *Differentiation* $\frac{d}{dx}[f(x)] = f'(x)$ $\xrightarrow{f'(x)}$ Pengamiran *Integration* $\int f'(x)\,dx = f(x)$; $\xleftarrow{f(x)}$

4. Secara am, kitab oleh tulis seperti berikut:
   In general, we can write as follows:

   Jika/ *If* $\frac{d}{dx}[f(x)] = f'(x)$, maka/ *hence* $\int f'(x)\,dx = f(x)$.

Video Tutorial

Pengamiran sebagai songsangan pembezaan
Integration as the inverse of differentiation

Info

MazeBab3

Video Tutorial

SP 3.1.1 Menjelaskan perkaitan antara pembezaan dan pengamiran.

1. Cari songsangan pembezaan yang berikut. TP 1
   Find the following inverse of differentiation.

Contoh

Diberi/*Given* $\frac{d}{dx}\left(\frac{1}{3}x^7\right) = \frac{7}{3}x^6$, cari/*find* $\int \frac{7}{3}x^6\,dx$.

Pembezaan bagi $\frac{1}{3}x^7$ ialah $\frac{7}{3}x^6$.

Differentiation of $\frac{1}{3}x^7$ *is* $\frac{7}{3}x^6$.

Jadi/*So*

$\int \frac{7}{3}x^6\,dx = \frac{1}{3}x^7$.

(a) Diberi/*Given* $\frac{d}{dx}(5x^4 + 3x) = 20x^3 + 3$, cari/*find* $\int (20x^3 + 3)\,dx$.

Pembezaan bagi $5x^4 + 3x$ ialah $20x^3 + 3$.
Differentiation of $5x^4 + 3x$ *is* $20x^3 + 3$.
Jadi/*So*
$\int (20x^3 + 3)\,dx = 5x^4 + 3x$

(b) Diberi/*Given* $\frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2}$, cari/*find* $\int \frac{1}{(1-x)^2}\,dx$.

Pembezaan bagi $\frac{x}{1-x}$ ialah $\frac{1}{(1-x)^2}$.

Differentiation of $\frac{x}{1-x}$ *is* $\frac{1}{(1-x)^2}$.
Jadi/*So*
$\int \frac{1}{(1-x)^2}\,dx = \frac{x}{1-x}$

(c) Diberi/*Given* $\frac{d}{dx}x(2x-1)^2 = (2x-1)(6x-1)$, cari/*find* $\int (2x-1)(6x-1)\,dx$.

Pembezaan bagi $x(2x-1)^2$ ialah $(2x-1)(6x-1)$.
Differentiation $x(2x-1)^2$ *is* $(2x-1)(6x-1)$.
Jadi/*So*
$\int (2x-1)(6x-1)\,dx = x(2x-1)^2$

KUASAI PBD FORMATIF

3.2 Kamiran Tak Tentu
Indefinite integral

Buku Teks ms. 85 – 91

KUASAI Nota Pintas

Untuk kamiran ax^n terhadap x.
Integrate ax^n with respect to x.

$$\int ax^n\,dx = \frac{ax^{n+1}}{(n+1)} + c,\ n \neq -1,$$ dengan a dan c ialah pemalar.
where a and c are constants.

Info
Kamiran ungkapan algebra
Integration of algebraic expressions

SP 3.2.1 Menerbitkan rumus kamiran tak tentu secara induktif.

2. Cari kamiran tak tentu yang berikut. TP 2
Find the following indefinite integral.

Contoh

$$\int \frac{-2}{3x^3}\,dx = \int \frac{-2}{3}x^{-3}dx$$
$$= \frac{-2x^{-3+1}}{3(-3+1)} + c$$
$$= \frac{1}{3x^2} + c$$

Tip Penting
Mesti tukar kepada bentuk ax^n.
Must convert to ax^n.

(a) $$\int \frac{\sqrt{x}}{4x^3}\,dx = \int \frac{x^{\frac{1}{2}-3}}{4}\,dx$$
$$= \int \frac{x^{-\frac{5}{2}}}{4}dx$$
$$= \frac{1}{4}\left(\frac{-2}{3}\right)x^{-\frac{3}{2}} + c$$
$$= \frac{-1}{6\sqrt{x^3}} + c$$

(b) $$\frac{3}{5}\int \frac{(5x)^2}{\sqrt{x}}\,dx = \frac{3}{5}\int \frac{25x^2}{x^{\frac{1}{2}}}\,dx$$
$$= \int 15x^{\frac{3}{2}}\,dx = 15\left(\frac{2}{5}\right)x^{\frac{5}{2}}$$
$$= 6x^{\frac{5}{2}} + c = 6\sqrt{x^5} + c$$

(c) $$\int \frac{px}{qx^{-4}}\,dx = \int \frac{px^5}{q}\,dx$$
$$= \frac{px^6}{6q} + c$$

SP 3.2.2 Menentukan kamiran tak tentu bagi fungsi algebra.

3. Tentukan kamiran tak tentu bagi fungsi algebra yang berikut. TP 2
Determine the indefinite integral for each of the following algebraic expression.

Contoh

$$\int \frac{x(x^2-4)}{3}\,dx$$
$$= \int \frac{x^3}{3}\,dx - \int \frac{4x}{3}\,dx$$
$$= \frac{x^4}{3(4)} - \frac{4x^2}{3(2)} + c$$
$$= \frac{x^4}{12} - \frac{2x^2}{3} + c$$

Tip Penting
Guna/*Use*.
$\int [f(x) \pm g(x)]dx$
$= \int f(x)dx \pm \int g(x)dx$

(a) $$\int \left(-3x^2 - \frac{2}{3}x + \frac{1}{x^2}\right)dx$$
$$= \int \left(-3x^2 - \frac{2}{3}x + x^{-2}\right)dx$$
$$= -x^3 - \frac{1}{3}x^2 - \frac{1}{x} + c$$

(b) $$\int 2x^2\left(1 + \frac{1}{x} - \frac{1}{3x^4}\right)dx$$
$$= \int \left(2x^2 + 2x - \frac{2}{3}x^{-2}\right)dx$$
$$= \frac{2}{3}x^3 + x^2 + \frac{2}{3x} + c$$

(c) $$\int \frac{3x^2+1}{\sqrt[3]{x^2}}\,dx$$
$$= \int 3x^{2-\frac{2}{3}} + x^{-\frac{2}{3}}\,dx$$
$$= \int 3x^{\frac{4}{3}} + x^{-\frac{2}{3}}\,dx$$
$$= \frac{9}{7}x^{\frac{7}{3}} + 3x^{\frac{1}{3}} + c$$

(d) $$\int (x+4)(3-x)dx$$
$$= \int (3x - x^2 + 12 - 4x)dx$$
$$= \int (-x^2 + 12 - x)dx$$
$$= \frac{-1}{3}x^3 + 12x - \frac{1}{2}x^2 + c$$

(e) $$\int \frac{5t(t^2-16)}{(t-4)}dt$$
$$= \int \frac{5t(t+4)(t-4)}{(t-4)}dt$$
$$= \int 5t^2 + 20t\,dt$$
$$= \frac{5}{3}t^3 + 10t^2 + c$$

SP 3.2.3 Menentukan kamiran tak tentu bagi fungsi berbentuk $(ax + b)^n$, dengan keadaan a dan b ialah pemalar, n integer dan $n \neq -1$.

4. Tentukan kamiran tak tentu bagi fungsi berbentuk $(ax + b)^n$, dengan keadaan a dan b ialah pemalar, dan $n \neq -1$. TP 3

Determine the indefinite integral for the function $(ax + b)^n$, where a and b are constants, and $n \neq -1$.

Contoh

$\int \frac{2}{3}(3x-2)^4 dx$

Katakan/*Let* $u = 3x - 2$

$\frac{du}{dx} = 3$, $dx = \frac{du}{3}$

$\int \frac{2}{3} u^4 \frac{du}{3} = \frac{2u^{4+1}}{9(4+1)} + c$

$= \frac{2(3x-2)^5}{45} + c$

(a) $\int \frac{a}{b}(3-4x)^3 dx$, a dan b ialah pemalar.

$\int \frac{a}{b}(3-4x)^3 dx$, a and b are constants.

Katakan/*Let* $u = 3 - 4x$

$\frac{du}{dx} = -4$, $dx = \frac{du}{-4}$

$\int \frac{a}{b} u^3 \frac{du}{-4}$

$= \frac{a(3-4x)^4}{-16b} + c$

(b) $\int \frac{6}{(1-3x)^4} dx$

Katakan/*Let* $u = 1 - 3x$

$\frac{du}{dx} = -3$, $dx = \frac{du}{-3}$

$\int 6u^{-4} \frac{du}{-3}$

$= \frac{-2u^{-4+1}}{(-3)} + c$

$= \frac{2}{3(1-3x)^3} + c$

(c) $\int \frac{1}{2}\sqrt{x+7}\, dx$

Katakan/*Let* $u = x + 7$

$\frac{du}{dx} = 1$, $dx = du$

$\int \frac{1}{2} u^{\frac{1}{2}} du = \frac{1u^{\frac{3}{2}}}{3} + c$

$= \frac{1}{3}(x+7)\sqrt{x+7} + c$

(d) $\int \frac{9}{5\left(2-\frac{1}{2}x\right)^3} dx$

Katakan/*Let* $u = 2 - \frac{1}{2}x$

$\frac{du}{dx} = -\frac{1}{2}$, $dx = -2\, du$

$\int \frac{9}{5} u^{-3}(-2)\, du = \frac{-18u^{-3+1}}{5(-2)} + c$

$= \frac{9}{5\left(2-\frac{1}{2}x\right)^2} + c$

(e) $\int \frac{4}{\sqrt{1-3x}} dx$

Katakan/*Let* $u = 1 - 3x$

$\frac{du}{dx} = -3$, $dx = \frac{-du}{3}$

$\int \frac{4}{-3} u^{-\frac{1}{2}} du = \frac{4u^{-\frac{1}{2}+1}}{-3\left(\frac{1}{2}\right)} + c$

$= \frac{-8}{3}\sqrt{1-3x} + c$

SP 3.2.4 Menentukan persamaan lengkung daripada fungsi kecerunan.

5. Tentukan persamaan lengkung, y daripada fungsi kecerunan yang berikut. TP 3

Determine the equation of the curve, y from the following gradient functions.

Contoh

Diberi fungsi kecerunan, $\frac{dy}{dx} = x^2 - 2$, $y = 1$ apabila $x = -3$.

Given the gradient function, $\frac{dy}{dx} = x^2 - 2$, $y = 1$ when $x = -3$.

$\frac{dy}{dx} = x^2 - 2$

$\int dy = \int (x^2 - 2)\, dx$

$y = \frac{1}{3}x^3 - 2x + c$

Apabila / *When* $x = -3$, $y = 1$

$1 = -9 + 6 + c$

$c = 4$

Persamaan lengkung ialah $y = \frac{1}{3}x^3 - 2x + 4$

The equation of the curve is

Tip Penting

Persamaan lengkung ialah fungsi y dalam sebutan x. Guna x dan y untuk mendapat nilai c.

The equation of curve is function y in terms of x. Use x and y to get value of c.

(a) Diberikan fungsi kecerunan ialah $\frac{1}{2} - x^2 - x$ dan $y = 2$ apabila $x = 0$.

Given the gradient function is $\frac{1}{2} - x^2 - x$ and $y = 2$ when $x = 0$.

$\frac{dy}{dx} = \frac{1}{2} - x^2 - x$

$\int dy = \int \left(\frac{1}{2} - x^2 - x\right) dx$

$y = \frac{1}{2}x - \frac{1}{3}x^3 - \frac{1}{2}x^2 + c$

Apabila/*When* $x = 0$, $y = 2$

$c = 2$

Persamaan lengkung ialah

The equation of the curve is

$y = \frac{1}{2}x - \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2$

(b) Diberi $\frac{dy}{dx} = \sqrt{2x+1}$ dan $y = 8$ apabila $x = 4$.

Given $\frac{dy}{dx} = \sqrt{2x+1}$ and $y = 8$ when $x = 4$.

$$\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$$
$$\int dy = \int (2x+1)^{\frac{1}{2}}\,dx$$
$$y = \frac{2}{3}\left(\frac{1}{2}\right)(2x+1)^{\frac{3}{2}} + c$$
$$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + c$$

Apabila/*When* $x = 4$, $y = 8$

$$8 = \frac{1}{3}(3^3) + c$$
$$c = -1$$

Persamaan lengkung ialah

The equation of the curve is

$$y = \frac{1}{3}(2x+1)^{\frac{3}{2}} - 1$$

(c) Diberi $\frac{dy}{dx} = (ax+5)$ dan graf melalui titik $(-1, 4)$ dan $(0, -2)$, dengan a ialah pemalar.

Given $\frac{dy}{dx} = (ax+5)$ and the graph passes through the point $(-1, 4)$ and $(0, -2)$, where a is a constant.

$$\frac{dy}{dx} = ax + 5$$
$$\int dy = \int (ax+5)\,dx$$
$$y = \frac{a}{2}x^2 + 5x + c$$

Apabila/*When* $x = -1$, $y = 4$

$$4 = \frac{1}{2}a - 5 + c$$
$$9 = \frac{1}{2}a + c \quad \ldots\ldots ①$$

Apabila/*When* $x = 0$, $y = -2$

$$-2 = c$$
$$a = 22$$

Persamaan lengkung ialah

The equation of the curve is

$$y = 11x^2 + 5x - 2$$

Cuba jawab **Praktis Sumatif 3**, *K1: S2*

Video Tutorial

KUASAI PBD FORMATIF 3.3 Kamiran Tentu *Definite Integral*

Buku Teks ms. 92 – 110

KUASAI Nota Pintas

Video Tutorial — *Kamiran tentu* *Definite integral*

Info — Petua Kamiran Tentu *Rules of Definite Integral*

1. Jika $\int f(x)dx = F(x) + c$, dan apabila $x = a$, maka $\int f(x)dx = F(a) + c$
   If $\int f(x)dx = F(x) + c$, and when $x = a$, then $\int f(x)dx = F(a) + c$
2. Pengamiran fungsi $f(x)$ terhadap x dari $x = a$ hingga $x = b$ ialah ditulis seperti
   The integration of function $f(x)$ with respect to x from $x = a$ to $x = b$ is written as
   $\int_a^b f(x)dx = [F(x) + c]_a^b = [F(b) + c] - [F(a) + c] = F[b] - F[a]$

SP 3.3.1 Menentukan nilai kamiran tentu bagi fungsi algebra.

6. Nilaikan yang berikut. **TP 3**

Evaluate the following.

Contoh

$$\int_{-1}^{3} (4-2x)(x+3)dx$$
$$\int_{-1}^{3} (-2x^2 - 2x + 12)dx$$
$$= \left[\frac{-2}{3}x^3 - x^2 + 12x\right]_{-1}^{3}$$
$$= [-18 - 9 + 36] - \left[\frac{2}{3} - 1 - 12\right]$$
$$= 21\frac{1}{3}$$

Tip Penting

Ganti nilai $x = 3$, ikut dengan $x = -1$. Semak nilai dari kalkulator.

Replace the value $x = 3$, follow by $x = -1$. Check the value with the calculator.

(a) $\int_{-1}^{0} 2x^2(x+6)dx$

$$\int_{-1}^{0} 2x^3 + 12x^2 dx$$
$$= \left[\frac{1}{2}x^4 + 4x^3\right]_{-1}^{0}$$
$$= \left[\frac{1}{2}(0)^4 + 4(0)^3\right] - \left[\frac{1}{2}(-1)^4 + 4(-1)^3\right]$$
$$= [0] - \left[\frac{1}{2} - 4\right] = 3\frac{1}{2}$$

(b) $\int_{-2}^{1} \frac{5x^3 - 2}{x^2}dx$

$$\int_{-2}^{1} 5x - 2x^{-2}dx$$
$$= \left[\frac{5}{2}x^2 + 2x^{-1}\right]_{-2}^{1}$$
$$= \left[\frac{5}{2} + 2\right] - [10 - 1] = -4\frac{1}{2}$$

(c) $\int_0^2 \frac{4}{5(1-2x)^{-2}}dx$

$\int_0^2 \frac{4}{5}(1-2x)^2 dx$

$= \frac{4}{5(3)(-2)}\left[(1-2x)^3\right]_0^2$

$= -\frac{2}{15}[(-27-1)] = \frac{56}{15}$

(d) $\int_{-2}^2 \frac{9x^2-16}{2(3x+4)}dx$

$= \int_{-2}^2 \frac{1}{2}(3x-4)dx$

$= \int_{-2}^2 \left(\frac{3}{2}x-2\right)dx$

$= \left[\frac{3}{4}x^2-2x\right]_{-2}^2$

$= (3-4)-(3+4) = -8$

7. Cari nilai yang berikut. TP 4

Find the value of the following.

Contoh

Diberi/*Given* $\int_{-3}^3 f(x)dx = 6$. $\int_0^3 f(x)dx = 4$, dan/*and* $\int_{-3}^3 g(x)dx = 2$, cari nilai bagi / *find the value of*

(i) $\int_3^{-3} 2f(x)dx$

(ii) $\int_{-3}^3 [g(x)-f(x)]dx$

(iii) $\int_{-3}^0 \frac{1}{2}f(x)dx$ jika/*if* $\int_0^3 f(x)dx = 4$

(iv) $\int_{-3}^1 3f(x)dx + \int_1^3 3f(x)dx$

(v) k jika/*if* $\int_{-3}^3 [k-g(x)]dx = 7$

(i) $\int_3^{-3} 2f(x)dx = 2(6) = -12$

(ii) $\int_{-3}^3 [g(x)-f(x)]dx$

$= \int_{-3}^3 g(x)dx - \int_{-3}^3 f(x)dx = 2-6 = -4$

(iii) $\int_{-3}^3 f(x)dx = \int_{-3}^0 f(x)dx + \int_0^3 f(x)dx$

$6 = \int_{-3}^0 f(x)dx + 4$

$\int_{-3}^0 f(x)dx = 2$

$\int_{-3}^0 \frac{1}{2}f(x)dx = \frac{1}{2}(2) = 1$

(iv) $\int_{-3}^1 3f(x)dx + \int_1^3 3f(x)dx$

$= 3\int_{-3}^3 f(x)dx$

$= 3(6)$

$= 18$

(v) $\int_{-3}^3 [k-g(x)]dx = 7$

$\int_{-3}^3 kdx - \int_{-3}^3 g(x) = 7$

$[kx]_{-3}^3 = 7+2$

$3k+3k = 9$

$k = \frac{9}{6} = \frac{3}{2}$

Tip Penting

(iv) $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

(v) Pisahkan kepada sebutan berasingan bergantung kepada had yang diberi

Split into different terms according to the limits provided.

(a) Diberi/*Given* $\int_{-2}^5 f(x)dx = 7$. $\int_{-2}^0 f(x)dx = -2$, dan/*and* $\int_{-2}^5 g(x)dx = 3$, cari nilai bagi/ *find the value of*

(i) $\int_5^{-2} f(x)dx$

(ii) $\int_{-2}^5 \left[\frac{1}{3}g(x) - \frac{1}{2}f(x)\right]dx$

(iii) $\frac{3}{2}\int_{-2}^0 f(x)dx + \int_0^5 \frac{3}{2}f(x)dx$

(iv) $\int_5^{-2} 2f(x)\,dx + \int_{-2}^0 \frac{1}{2}f(x)dx$

(v) k jika/*if* $\int_{-2}^5 [2g(x)-kx]dx = 5$

(i) $\int_5^{-2} f(x)dx = -7$

(ii) $\int_{-2}^5 \frac{1}{3}g(x)dx - \int_{-2}^5 \frac{1}{2}f(x)dx$

$= \frac{1}{3}(3) - \frac{1}{2}(3) = -\frac{1}{2}$

(iii) $\frac{3}{2}\int_{-2}^0 f(x)\,dx + \int_0^5 \frac{3}{2}f(x)dx$

$= \frac{3}{2}\int_{-2}^5 f(x)dx = \frac{21}{2}$

(iv) $\int_5^{-2} 2f(x)\,dx + \int_{-2}^0 \frac{1}{2}f(x)dx$

$= 2(-7) + \frac{1}{2}(-2) = -15$

(v) $\int_{-2}^5 [2g(x)-kx]dx = 5$

$\int_{-2}^5 2g(x)\,dx - \int_{-2}^5 kxdx = 5$

$6-5 = \int_{-2}^5 kxdx$

$1 = \left[\frac{k}{2}x^2\right]_{-2}^5$

$\frac{2}{k} = 25-4$

$k = \frac{2}{21}$

Cuba jawab **Praktis Sumatif 3**, *K1: S5*

8. Selesaikan yang berikut. TP 4
Solve the following.

Contoh

Diberi $\int_{-1}^{k}(x-4)dx = -12$, cari nilai k yang mungkin.
Given $\int_{-1}^{k}(x-4)dx = -12$, *find the possible value of k.*

$$\int_{-1}^{k}(x-4)dx = -12$$
$$\left[\frac{1}{2}x^2 - 4x\right]_{-1}^{k} = -12$$
$$\frac{1}{2}k^2 - 4k - \left[\frac{1}{2} + 4\right] = -12$$
$$\frac{1}{2}k^2 - 4k + \frac{15}{2} = 0$$
$$k^2 - 8k + 15 = 0$$
$$(k-3)(k-5) = 0$$
$k = 3$ atau/*or* 5

(a) Diberi $\int_{k}^{2}(2-3x)dx = 8$, cari nilai k, dengan keadaan $k < 0$.
Given $\int_{k}^{2}(2-3x)\,dx = 8$, *find the value of k, where* $k < 0$.

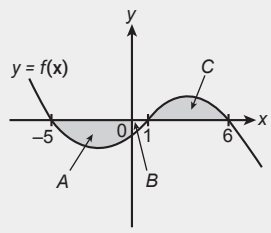
$$\int_{k}^{2}(2-3x)\,dx = 8$$
$$\left[2x - \frac{3}{2}x^2\right]_{k}^{2} = 8$$
$$[4-6] - [2k - \frac{3}{2}k^2] = 8$$
$$\frac{3}{2}k^2 - 2k - 10 = 0$$
$$3k^2 - 4k - 20 = 0$$
$$(3k-10)(k+2) = 0$$
$k = \frac{10}{3}$ atau/*or* -2
Maka/*Hence* $k = -2$

SP 3.3.2 Menyiasat dan menerangkan perkaitan antara had bagi hasil tambah luas segi empat tepat dengan luas di bawah suatu lengkung.

9. Cari kamiran tentu atau luas berdasarkan rajah yang diberi. TP 4
Find the value of the definite integral or area based on the diagram given.

Info
Luas rantau di bawah graf
Area under the graph

Contoh



Diberi luas rantau A ialah 8 unit², luas B ialah 2 unit² dan $\int_{-5}^{6} f(x)dx = 1$, cari
Given the area of the region A is 8 unit², the area of B is 2 unit² and $\int_{-5}^{6} f(x)dx = 1$, *find*

(i) $\int_{-5}^{1} f(x)\,dx$
(ii) $\int_{1}^{0} f(x)\,dx$
(iii) $\int_{1}^{6} f(x)\,dx$

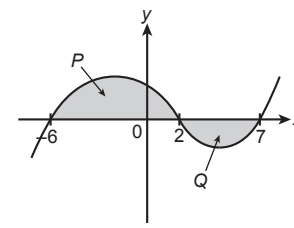
Tip Penting
Luas di bawah paksi-x ialah negatif.
The area under the x-axis is negative.

(i) $\int_{-5}^{1} f(x)\,dx = \int_{-5}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$
$= -8 + (-2)$
$= -10$

(ii) $\int_{1}^{0} f(x)\,dx = -\int_{0}^{1} f(x)\,dx = -(-2)$
$= 2$

(iii) $\int_{-5}^{1} f(x)\,dx + \int_{1}^{6} f(x)\,dx = \int_{-5}^{6} f(x)\,dx$
$\int_{1}^{6} f(x)\,dx = \int_{-5}^{6} f(x)\,dx - \int_{-5}^{1} f(x)\,dx$
$= 1 - (-10)$
$= 11$

(a)



Diberi luas P ialah 6.8 unit² dan $\int_{0}^{2} f(x)\,dx = 1.5$, dan luas Q ialah 4.2 unit², cari
Given the area of P is 6.8 unit² and $\int_{0}^{2} f(x)\,dx = 1.5$, *and the area of Q is 4.2 unit², find*

(i) $\int_{0}^{7} f(x)\,dx$
(ii) $\int_{-6}^{0} f(x)\,dx$
(iii) $\int_{-6}^{7} f(x)\,dx$
(iv) jumlah luas P dan Q
the total area of P and Q

(i) $\int_{0}^{7} f(x)\,dx = \int_{0}^{2} f(x)\,dx + \int_{2}^{7} f(x)\,dx$
$= 1.5 + (-4.2)$
$= -2.7$

(ii) $\int_{-6}^{0} f(x)\,dx = \int_{-6}^{2} f(x)\,dx - \int_{0}^{2} f(x)\,dx$
$= 6.8 - 1.5$
$= 5.3$

(iii) $\int_{-6}^{7} f(x)\,dx = \int_{-6}^{0} f(x)\,dx + \int_{0}^{7} f(x)\,dx$
$= 5.3 + (-2.7)$
$= 2.6$

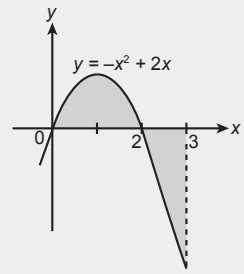
(iv) Jumlah luas P dan Q $= 6.8 + 4.2$
Total area of P and Q $= 11$ unit²

SP 3.3.3 Menentukan luas suatu rantau

10. Cari luas rantau berlorek bagi rajah yang berikut dengan paksi-x. TP 4
Find the area of the shaded region for each of the following diagrams with the x-axis.

Contoh

$y = -x^2 + 2x$



$y = -x^2 + 2x$
$= x(-x + 2)$

Luas rantau berlorek
Area of the shaded region

$= \int_0^2 f(x)\,dx + \left|\int_2^3 f(x)\,dx\right|$

$= \int_0^2 (-x^2 + 2x)\,dx + \left|\int_2^3 (-x^2 + 2x)\,dx\right|$

$= \left[-\frac{1}{3}x^3 + x^2\right]_0^2 + \left|\left[-\frac{1}{3}x^3 + x^2\right]_2^3\right|$

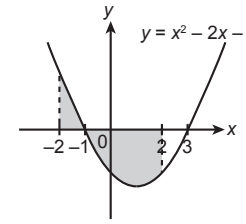
$= \left(-\frac{8}{3} + 4\right) + \left|(-9 + 9) - \left(-\frac{8}{3} + 4\right)\right|$

$= 1\frac{1}{3} + 1\frac{1}{3} = 2\frac{2}{3}$ unit^2

Tip Penting

Perlu membuat pengamiran berasingan kerana sebahagian luas di atas paksi-x dan sebahagian luas di bawah paksi-x.
Jawapan $\int_0^3 (-x^2 + 2x)\,dx$ ialah 0.
We need to integrate separately because one part of the area is above x-axis and the other one below x-axis. $\int_0^3 (-x^2 + 2x)\,dx$ *is 0.*

(a) $y = x^2 - 2x - 3$



Luas rantau berlorek/*Area of the shaded region*

$= \int_{-2}^{-1} (x^2 - 2x - 3)\,dx + \left|\int_{-1}^{2} (x^2 - 2x - 3)\,dx\right|$

$= \left[\frac{1}{3}x^3 - x^2 - 3x\right]_{-2}^{-1} + \left|\left[\frac{1}{3}x^3 - x^2 - 3x\right]_{-1}^{2}\right|$

$= \left(-\frac{1}{3} - 1 + 3\right) - \left(-\frac{8}{3} - 4 + 6\right) + \left|\left(\frac{8}{3} - 4 - 6\right) - \left(-\frac{1}{3} - 1 + 3\right)\right|$

$= 2\frac{1}{3} + 9$

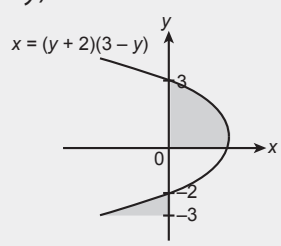
$= 11\frac{1}{3}$ unit^2

Cuba jawab **Praktis Sumatif 3**, *K2: S1*

11. Cari luas rantau berlorek bagi rajah yang berikut dengan paksi-y. TP 4
Find the area of the shaded region with the y-axis for the following diagrams.

Contoh

$x = (y + 2)(3 - y)$



Luas rantau berlorek/*Area of the shaded region*

$= \left|\int_{-3}^{-2} (-y^2 + y + 6)dy\right| + \int_0^3 (-y^2 + y + 6)dy$

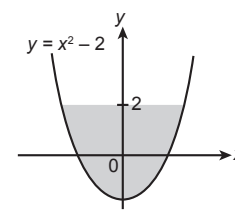
$= \left[-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 6y\right]_{-3}^{-2} + \left[-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 6y\right]_0^3$

$= \left|\left[\left(\frac{8}{3} + 2 - 12\right) - \left(\frac{27}{3} + \frac{9}{2} - 18\right)\right|\right.$
$\quad + \left(-9 + \frac{9}{2} + 18\right)$

$= \left|-2\frac{5}{6}\right| + 13\frac{1}{2}$

$= 16\frac{1}{3}$ unit^2

(a) $y = x^2 - 2$



Apabila/*When* $x = 0$, $y = -2$

Luas rantau berlorek
Area of the shaded region

$= 2\int_{-2}^{2} x\,dy$

$= 2\int_{-2}^{2} [(y + 2)^{\frac{1}{2}}\,dy$

$= 2\left[\frac{2}{3}(y + 2)^{\frac{3}{2}}\right]_{-2}^{2}$

$= 2\left[\left(\frac{16}{3} - 0\right)\right]$

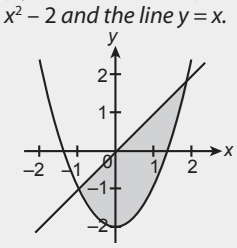
$= \frac{32}{3}$ unit^2

Cuba jawab **Praktis Sumatif 3**, *K2: S1*

12. Cari luas yang dibatasi oleh kedua-dua graf yang diberi bagi yang berikut. TP 4
Find the area bounded by the two graphs for each of the following.

Contoh

Diberi lengkung $y = x^2 - 2$ dan garis lurus $y = x$.
Given the curve $y = x^2 - 2$ and the line $y = x$.



Graf adalah minimum pada (0, –2).
Graph is minimum at (0, –2).

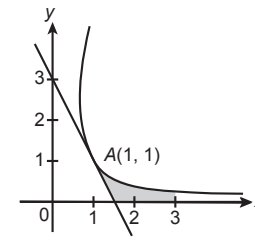
Titik persilangan $x^2 - 2 = x$
Intersection point $x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1, 2$

Luas dibatasi $= \int_{-1}^{2} [f(x) - g(x)]dx$
Bounded area
$= \int_{-1}^{2} [x - (x^2 - 2)]\, dx$
$= \int_{-1}^{2} [x - x^2 + 2]dx$
$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right]$
$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$
$= 4.5$ unit^2

Tip Penting

Perlu cari titik persilangan. / *Find the point of intersection*
Guna $\int_a^b f(x)dx - \int_a^b g(x)dx$ jika luas dibatasi oleh kedua-dua graf sahaja.
Use $\int_a^b f(x)dx - \int_a^b g(x)dx$ if the area bounded by both graphs only.

(a) Diberi graf lengkung $y = \frac{1}{x^2}$ dan tangen pada titik (1, 1)
Given the equation of the curve $y = \frac{1}{x^2}$ and tangent at point (1, 1).



$y = x^{-2}$, $\frac{dy}{dx} = -2x$

Di A(1, 1), persamaan tangen ialah $y - 1 = -2(x - 1)$.
At A(1, 1), the equation of tangent is $y - 1 = -2(x - 1)$.

$y = -2x + 3$

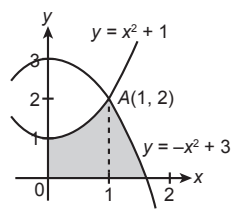
Apabila/*When* $y = 0$, $x = \frac{3}{2}$

Luas di bawah lengkung $y = \frac{1}{x^2}$
Area under the curve
$= \int_1^3 x^{-2}\, dx = [-x^{-1}]_1^3$
$= \frac{2}{3}$ unit^2

Luas di bawah tangen $= \frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4}$ unit^2
Area under the tangent

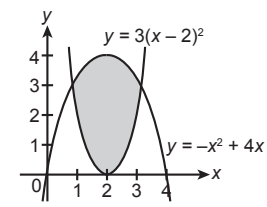
Maka, luas berlorek $= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ unit^2
Hence, the shaded area

(b) Diberi lengkung $y = x^2 + 1$ dan $y = -x^2 + 3$. Cari luas dibatasi oleh kedua-dua lengkung, paksi-x dan paksi-y.
Given the curves $y = x^2 + 1$ and $y = -x^2 + 3$. Find the area bounded by the two curves, the x and y axes.



Titik persilangan, $x^2 + 1 = -x^2 + 3$
Intersection point $2x^2 = 2$, $x = 1$
Titik persilangan dengan paksi-x ialah $\sqrt{3}$.
Intersection point with x-axis is
Luas dibatasi oleh kedua-dua graf
Area bounded by the two curves
$= \int_0^1 (x^2 + 1)]\, dx + \int_1^{\sqrt{3}} (-x^2 + 3)]\, dx$
$= \left[\frac{1}{3}x^3 + x\right]_0^1 + \left[-\frac{1}{3}x^3 + 3x\right]_1^{\sqrt{3}}$
$= 1\frac{1}{3} + 2\sqrt{3} - \left(2\frac{2}{3}\right)$
$= \left(2\sqrt{3} - \frac{4}{3}\right)$ unit^2

(c) Diberi lengkung $y = 3(x - 2)^2$ dan $y = x(4 - x)$.
Given the curves $y = 3(x - 2)^2$ and $y = x(4 - x)$.



Titik persilangan: $3(x - 2)^2 = -x^2 + 4x$
Intersection point: $x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1$ dan/*and* 3

Luas dibatasi oleh dua lengkung ialah
Area bounded by two curves is
$= \int_1^3 [f(x) - g(x)]dx$
$= \int_1^3 [4x - x^2 - (3(x - 2)^2]\, dx$
$= \int_1^3 [16x - 4x^2 - 12]dx$
$= \left[8x^2 - \frac{4}{3}x^3 - 12x\right]_1^3$
$= \left[(72 - 36 - 36) - \left(8 - \frac{4}{3} - 12\right)\right]$
$= 5\frac{1}{3}$ unit^2

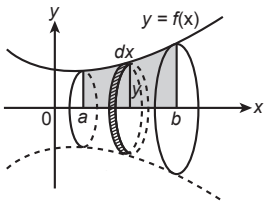
Cuba jawab **Praktis Sumatif 3**, *K2: S5*

Video Tutorial

Isi padu janaan
The generated volume

KUASAI Nota Pintas

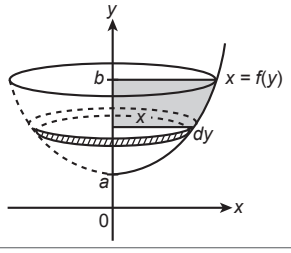
(a)



Apabila kawasan berlorek dalam rajah diputarkan melalui 360° pada paksi-*x*, isi padu dijanakan dalam notasi pengamiran ialah
When the shaded region in the diagram is rotated through 360° about the x-axis, the volume of revolution in integration notation is

$I = \pi \int_a^b y^2\, dx$ atau/*or* $I = \pi \int_a^b [f(x)]^2\, dx$

(b)



Apabila kawasan berlorek dalam rajah diputarkan melalui 360° pada paksi-*y*, isi padu dijanakan dalam notasi pengamiran ialah
When the shaded region in the diagram is rotated through 360° about the y-axis, the volume of revolution in integration notation is

$I = \pi \int_a^b x^2\, dy$ atau/*or* $I = \pi \int_a^b [f(y)]^2\, dy$

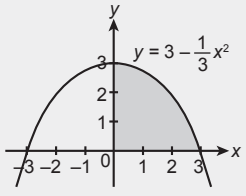
Video Tutorial

SP 3.3.4 Menyiasat dan menerangkan perkaitan antara had bagi hasil tambah isi padu silinder dengan isi padu janaan daripada kisaran suatu rantau.

13. Tentukan isi padu janaan dalam sebutan π apabila luas berlorek itu diputarkan melalui 360° pada paksi-*x*.
Determine the generated volume, in terms of π when the shaded region is rotated through 360° about the x-axis. TP 4

Contoh

Diberi persamaan $y = 3 - \frac{1}{3}x^2$
Given the equation



Guna isi padu dijanakan /*Use the generated volume*

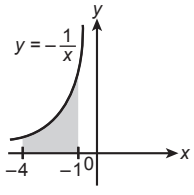
Isi padu janaan
The generated volume

$$= \pi\int_0^3 \left(3 - \frac{1}{3}x^2\right)^2 dx$$
$$= \pi\int_0^3 \left(9 - 2x^2 + \frac{1}{9}x^4\right) dx$$
$$= \pi\left[9x - \frac{2}{3}x^3 + \frac{1}{45}x^5\right]_0^3$$
$$= \pi\left(27 - 18 + \frac{27}{5}\right)$$
$$= 14\frac{2}{5}\pi \text{ unit}^3$$

Tip Penting

Lihat ilustrasi dari laman web di bawah.
See the illustration from the website.

(a) Diberi persamaan $y = \frac{-1}{x}$
Given the equation

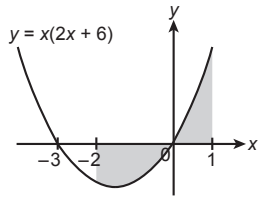


Guna isi padu dijanakan $= \pi\int_{-4}^{-1} y^2 dx$
Use the generated volume

Isi padu janaan
The generated volume

$$= \pi\int_{-4}^{-1}\left(\frac{1}{x^2}\right)dx$$
$$= \pi\left[\frac{-1}{x}\right]_{-4}^{-1}$$
$$= \pi\left(1 - \frac{1}{4}\right)$$
$$= \frac{3}{4}\pi \text{ unit}^3$$

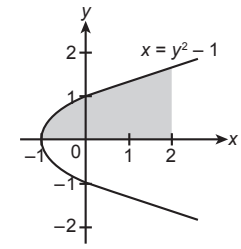
(b) Diberi persamaan $y = x(2x + 6)$
Given the equation



Isi padu janaan
The generated volume

$$= \pi\int_{-2}^{1} (x(2x+6))^2 dx$$
$$= \pi\int_{-2}^{1} (4x^4 + 24x^3 + 36x^2) dx$$
$$= \pi\left[\frac{4}{5}x^5 + 6x^4 + 12x^3\right]_{-2}^{1}$$
$$= \pi\left(\frac{4}{5} + 6 + 12\right) - \left(-\frac{128}{5} + 96 - 96\right)$$
$$= 44\frac{2}{5}\pi \text{ unit}^3$$

(c) Diberi persamaan $x = y^2 - 1$
Given the equation



Isi padu janaan
The generated volume

$$= \pi\int_{-1}^{2} y^2 dx$$
$$= \pi\int_{-1}^{2} (x+1) dx$$
$$= \pi\left[\frac{1}{2}x^2 + x\right]_{-1}^{2}$$
$$= \pi\left(\left[\frac{1}{2}(2)^2 + 2\right] - \left[\frac{1}{2}(-1)^2 + (-1)\right]\right)$$
$$= \pi\left[(2+2) - \left(\frac{1}{2} - 1\right)\right]$$
$$= \frac{9}{2}\pi \text{ unit}^3$$

14. Lakukan aktiviti berikut. TP 6
Carry out the following activity.

AKTIVITI **PAK-21**

Berkumpulan

Tujuan : Memahami dan menyiasat isi padu dijanakan apabila luas berlorek diputarkan pada paksi-y
Aim : *To understand and investigate the volume of revolution when a shaded area is rotated about the y-axis*

Langkah:

1. Imbas kod QR bagi mendapat simulasi perubahan nilai *A* dan *B* kepada isi padu yang dijanakan.
Scan the QR code for a simulation to change the value of A and B to get the different volumes of revolution.

2. Setiap kumpulan akan melakukan setiap aktiviti berikut.
Each group is to do the following activities.

(a) Seret gelongsor *A* supaya batasan bawah untuk $y = a$ boleh diubah
Slide the slider A to change the lower limit for y = a.

(b) Seret gelongsor B supaya batasan atas untuk $y = b$ boleh diubah
Slide the slider B to change the upper limit for y = b.

(c) Putarkan luas dengan menyeret gelongsor, perhatikan isi padu yang dijanakan.
Rotate the area by sliding the slider, observe the volume of revolution formed.

3. Jika ingin lihat fungsi yang lain, klik *algebra* dan cari bahagian '*function*'. Klik pada *f(x)*, dan masuk fungsi seperti $2x^2$ atau $\frac{1}{3}x^2 - 1$, dan seterusnya.
If you want to see the other functions, click algebra and look for the part labelled "function". Click on f(x), and key in the function like $2x^2$ *or* $\frac{1}{3}x^2 - 1$*, and so on*

4. Perhatikan isi padu janaan yang dibentuk daripada luas yang terpilih.
Observe the volume of revolution formed by the chosen area.

AKTIVITI PAK-21

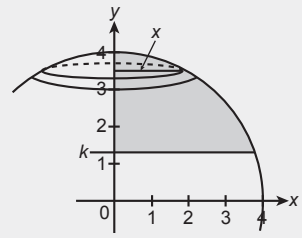
15. Tentukan nilai k dengan diberikan isi padu dalam sebutan π yang dijanakan apabila luas berlorek diputarkan melalui 360^0 pada paksi-y. TP 5

Determine the value of k for the given volume of revolution in terms of π when the shaded area is rotated through 360° about the y-axis.

Contoh

Diberi isi padu dijanakan oleh rantau berlorek ialah $13\frac{1}{3}\pi$ unit³ dengan graf $y^2 + x^2 = 16$.

Given the volume generated by the shaded region is $13\frac{1}{3}\pi$ unit³ by the curve $y^2 + x^2 = 16$.



Apabila/*When* $x = 0, y = 4$

Isi padu janaan $= \pi\int_k^4 x^2\,dy = 13\frac{1}{3}\pi$

The generated volume

$$\int_k^4 (16 - y^2)\,dy = \frac{40}{3}$$

$$\left[16y - \frac{1}{3}y^3\right]_k^4 = \frac{40}{3}$$

$$\left(64 - \frac{64}{3}\right) - \left(16k - \frac{1}{3}k^3\right) = \frac{40}{3}$$

$$\frac{1}{3}k^3 - 16k + \frac{88}{3} = 0$$

$$k^3 - 48k + 88 = 0$$

Cuba/*Try* $k = 2$, $2^3 - 48 \times 2 + 88 = 0$

Maka/*Hence* $(k - 2)(k^2 + 2k - 44) = 0$

$$k = 2,\ 5.7, -7.7$$

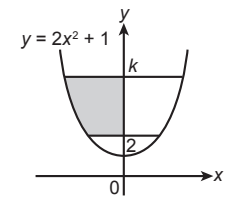
Tip Penting

Nilai k yang lain tidak sesuai.

Other values of k are not suitable.

(a) Diberi isi padu janaan ialah 2π unit³ oleh graf $y = 2x^2 + 1$ untuk $y = 2$ ke $y = k$.

Given the volume generated is 2π unit³ by the curve $y = 2x^2 + 1$ for $y = 2$ to $y = k$.



Isi padu janaan $= \pi\int_2^k x^2 dy = 2\pi$

The generated volume

$$\int_2^k \left(\frac{1}{2}y - \frac{1}{2}\right) dy = 2$$

$$\left[\frac{1}{4}y^2 - \frac{1}{2}y\right]_2^k = 2$$

$$\frac{1}{4}k^2 - \frac{1}{2}k - (1 - 1) = 2$$

$$k^2 - 2k - 8 = 0$$

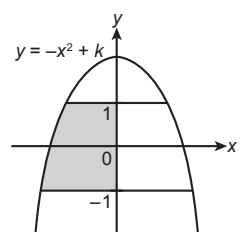
$$(k - 4)(k + 2) = 0$$

$$k = 4 \text{ atau/or } -2$$

Maka/*Hence* $k = 4$

(b) Diberi isi padu dijanakan oleh rantau berlorek ialah 4π unit³ dengan graf $y = -x^2 + k$.

Given the volume generated by the shaded region is 4π unit³ by the curve $y = -x^2 + k$.



Isi padu janaan $= \pi\int_{-1}^1 x^2 dy = 4\pi$

The generated volume

$$\int_{-1}^1 (k - y)dy = 4$$

$$\left[ky - \frac{1}{2}y^2\right]_{-1}^1 = 4$$

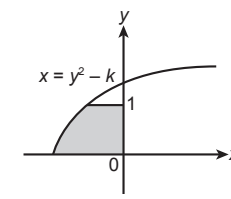
$$k - \frac{1}{2}k^2 - \left(-k - \frac{1}{2}k^2\right) = 3$$

$$2k = 4$$

$$k = 2$$

(c) Diberi isi padu dijanakan oleh rantau berlorek ialah $\frac{8}{15}\pi$ unit³ dengan graf $x = y^2 - k$ dari $y = 0$ ke $y = 1$.

Given the volume generated by the shaded region is $\frac{8}{15}\pi$ unit³ by the graph $x = y^2 - k$ from $y = 0$ to $y = 1$.



Isi padu janaan $= \pi\int_0^1 x^2 dy = \frac{8}{15}\pi$

The generated volume

$$\int_0^1 (y^2 - k)^2 dy = \frac{8}{15}$$

$$\int_0^1 (y^4 - 2ky^2 + k^2)dy = \frac{8}{15}$$

$$\left[\frac{1}{5}y^5 - \frac{2}{3}ky^3 + yk^2\right]_0^1 = \frac{8}{15}$$

$$\frac{1}{5} - \frac{2}{3}k + k^2 - (0) = \frac{8}{15}$$

$$3k^2 - 2k - 1 = 0$$

$$(3k + 1)(k - 1) = 0$$

$$k = \frac{-1}{3},\ 1$$

$k = \frac{-1}{3}$ (tidak diterima), maka $k = 1$

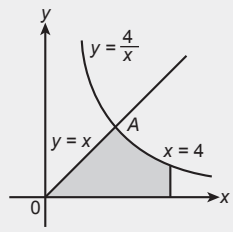
$k = \frac{-1}{3}$ *(not accepted), hence* $k = 1$

SP 3.3.5 Menentukan isi padu janaan bagi suatu rantau yang dikisarkan pada paksi-*x* atau paksi-*y*.

16. Tentukan isi padu janaan yang melibatkan lebih daripada satu lengkung bagi yang berikut. TP 6
Determine the generated volume involving more than one curve for the following.

Contoh

Luas dibatasi oleh paksi-*x*, lengkung $y = \frac{4}{x}$, $x = 4$ dan $y = x$ diputarkan melalui 360° pada paksi-*x*.
The area bounded by the x-axis, the curve $y = \frac{4}{x}$, $x = 4$ and $y = x$ is rotated through 360° about the x-axis.



$y = \frac{4}{x} = x$

$x^2 = 4$

$x = 2, y = 2$

Maka, titik $A(2, 2)$
Hence, point $A(2, 2)$

Perlu mencari titik persilangan dahulu.
First, need to find the intersection point.

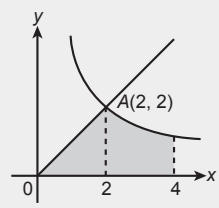
Jika garis *OA* diputarkan pada paksi-*x*, kita mendapat sebuah kon dengan jejari tapak 2 dan tinggi 2, iaitu isi padu kon $= \frac{1}{3}\pi(2)^2 2 = \frac{8}{3}\pi$ unit³
If the line OA is rotated about the x-axis, a cone of base radius 2 and height of 2 is obtained, that is
the volume of cone $= \frac{1}{3}\pi(2)^2 2 = \frac{8}{3}\pi$ *unit³*

Dari/*From* $x = 2$ ke/*to* $x = 4$.
Isi padu dijanakan oleh lengkung
Volume generated by the curve

$= \pi\int_2^4 y^2 dx$

$= \pi\int_2^4 \frac{16}{x^2} dx$

$= \pi\left[\frac{-16}{x}\right]_2^4 = 4\pi$ unit³

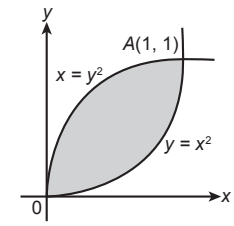


Jumlah isi padu $= \left(\frac{8}{3} + 4\right)\pi$
Total volume
$= 6\frac{2}{3}\pi$ unit³

Tip Penting

Sentiasa mencari titik persilangan dahulu bagi graf yang diberi. Selepas itu, lukis garis serenjang melalui titik persilangan kepada paksi-*x* supaya boleh membahagikan rantau kepada bahagian untuk pengamiran.
Always find the point of intersection of the given graphs. Then, draw a vertical line from the point of intersection to the x-axis in order to divide the region into sections for integration.

(a) Luas dibatasi oleh lengkung $y = x^2$ dan $x = y^2$ diputarkan melalui 360° pada paksi-*x*.
The area bounded by the curves $y = x^2$ and $x = y^2$ is rotated through 360° about the x-axis.



Persamaan lengkung $y = x^2$ …①
The equation of the curve

Persamaan lengkung $x = y^2$ …②
The equation of the curve

Gantikan ① ke dalam ②.
Substitute ① into ②.

$x = (x^2)^2$

$x = x^4$

Titik persilangan $x = x^4$
Point of intersection

$x(1 - x^3) = 0$

$x = 0$ dan/*and* $x = 1$

$A(1, 1)$

Isi padu janaan $= \pi\int_0^1 (\sqrt{x})^2 dx - \pi\int_0^1 x^4 dx$
The generated volume

$= \pi\left\{\left[\frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1\right\}$

$= \left(\frac{1}{2} - \frac{1}{5}\right)\pi$

$= \frac{3}{10}\pi$ unit³

Cuba jawab **Praktis Sumatif 3**, *K2: S2*

KUASAI PBD FORMATIF

3.4 Aplikasi Pengamiran
Applications of Integration

Buku Teks ms. 111 – 113

SP 3.4.2 Menyelesaikan masalah yang melibatkan pengamiran.

Video Tutorial
Aplikasi Pengamiran
Applications of Integration

17. Selesaikan masalah yang berikut. TP 6
Solve the following problems.

Contoh

Ram boleh mengecat pagar dengan kadar $(300 - 6t)$ m^2 setiap jam, dengan keadaan t ialah masa dalam jam sejak dia bermula. Jika pagar itu ialah 900 m^2, cari masa diambil untuk menghabiskan catnya. Beri jawapan betul kepada 1 tempat perpuluhan.
Ram is painting a fence at a rate of (300 – 6t) m^2 *per hour, where t is the number of hours since he started painting. If the fence is* 900 m^2, *find the time taken to finish the painting. Give the answer correct to 1 decimal then.*

Katakan luas pagar ialah L, dan masa diambil untuk menghabiskan cat ialah T, maka
Let say the area of the fence is L, and the time taken to finish the painting is T, then

$$\frac{dL}{dt} = 300 - 6t$$

$$\int_0^T \frac{dL}{dt}dt = \int_0^T (300 - 6t)dt = 900$$

$$[300t - 3t^2]_0^T = 900$$

$$3T^2 - 300T + 900 = 0$$

$$T^2 - 100T + 300 = 0$$

$$T = \frac{100 \pm \sqrt{100^2 - 4(300)}}{2}$$

$$= 96.9 \text{ jam atau } 3.1 \text{ jam}$$

Maka, masa diambil ialah 3.1 jam.
Hence, the time taken is 3.1 *hours.*

Tip Penting
96.9 jam ialah tidak munasabah.
96.9 hours is not reasonable.

(a) Kedalaman air di dalam tangki berubah dengan kadar $0.2t$ cm seminit, dengan keadaan t ialah masa dalam minit. Pada masa $t = 0$, kedalaman air ialah 25 cm.
The depth of water in a tank is changing at a rate of 0.2t cm *per minute, where t is the time in minutes. At time t* = 0, *the depth of the water is* 25 *cm.*

(i) Apakah perubahan dalam kedalamannya pada minit keempat?
What is the change of the depth in the fourth minute?

(ii) Apakah kedalaman air di dalam tangki apabila $t = 10$ minit?
What is the depth of water in the tank when t = 10 *minutes?*

(i) Katakan kedalaman air ialah h, maka
Let say the depth of water is h, hence

$$\frac{dh}{dt} = 0.2t$$

$$\int_3^4 \frac{dh}{dt}dt = \int_3^4 0.2t\, dt$$

$$h = [0.1t^2]_3^4$$

$$= 0.7 \text{ cm/minit}$$

(ii) $h = \int 0.2t\, dt = 0.1t + c$

Apabila/*When* $t = 0$, $h = 25$ cm

$$h = 0.1t + 25$$

Apabila/*When* $t = 10$ minit,

$$h = 0.1(10) + 25$$

$$= 26 \text{ cm}$$

Video Tutorial

KERTAS 1

1. Diberi/*Given* $\frac{d}{dx}[g(x)] = f(x)$, cari/*find* $\int f(x)dx$: SP 3.1.1

[2 *markah* / 2 *marks*]

Jawapan / *Answer*:

$\int f(x)dx = g(x)$

2. (a) Nilaikan / *Evaluate.* SP 3.3.1

$\int_0^1 \frac{2}{3(1-2x)^3}\,dx$

[2 *markah* / 2 *marks*]

(b) Diberi $\frac{dy}{dx} = (ax + 5)$ dan graf melalui titik (–1, 4) dan (0, –2), dengan keadaan a ialah pemalar. Cari SP 3.2.4

Given $\frac{dy}{dx} = (ax + 5)$ *and the graph passes through the point* (–1, 4) *and* (0, –2), *such that a is a constant. Find*

(i) nilai a.
the value of a.

[2 *markah* / 2 *marks*]

(ii) persamaan lengkung.
the equation of the curve.

[1 *markah* / 1 *mark*]

Jawapan / *Answer*:

(a) $\int_0^1 \frac{2}{3}(1-2x)^{-3}\,dx$

$= \frac{2}{3(-2)(-2)}[(1-2x)^{-2}]_0^1$

$= \frac{1}{6}[1-1]$

$= 0$

(b) (i) $\frac{dy}{dx} = ax + 5$

$\int dy = \int (ax+5)dx$

$y = \frac{a}{2}x^2 + 5x + c$

Apabila/*When* $x = -1, y = 4$

$4 = \frac{1}{2}a - 5 + c$

$9 = \frac{1}{2}a + c$ ..... ①

Apabila/*When* $x = 0, y = -2$

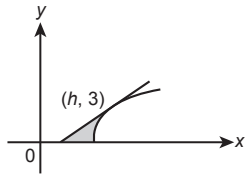
$-2 = c$

$a = 22$

(ii) Persamaan lengkung
The equation of the curve

$y = 11x^2 + 5x - 2$

3. Rajah menunjukkan garis lurus menyilang paksi-x pada $x = 1$ dan menyentuh lengkung pada (h, 3).
The diagram shows a straight line intersects the x-axis at x = 1 and touches the curve at (h, 3).



Diberi bahawa luas kawasan berlorek ialah 9 unit^2. Dengan menggunakan kaedah pengamiran, cari nilai h.
It is given that the area of the shaded region is 9 units. By using the method of integration, find the value of h. SP 3.3.3

[5 *markah* / 5 *marks*]

Jawapan / *Answer*:

Luas kawasan berlorek / *Area of the shaded region*

$= \int_0^3 f(y)dy - \frac{1}{2}[1+h]3 = 9$

$\int_0^3 (y^2 + 10)dy - \frac{1}{2}[1+h]3 = 9$

$\left[\frac{y^3}{3} + 10y\right]_0^3 - \frac{3}{2} - \frac{3}{2}h = 9$

$39 - 9 - \frac{3}{2} = \frac{3}{2}h$

$\frac{57}{2} = \frac{3}{2}h$

$h = 19$

4. Diberi $g(x) = \frac{6}{(1-2x)^3}$ dan $g'(x) = p(1-2x)^q$.

It is given that $g(x) = \frac{6}{(1-2x)^3}$ *and* $g'(x) = p(1-2x)^q$. SP 3.3.2

(a) Cari nilai bagi $p + q$.
Find the value of p + q.

[2 *markah* / 2 *marks*]

(b) Seterusnya, cari nilai m jika $\frac{1}{m}\int g'(x)dx = \frac{-2}{81}$ dan $x = 2$.
Subsequently, find the value of m if $\frac{1}{m}\int g'(x)dx = \frac{-2}{81}$ *and x = 2.*

[3 *markah* / 3 *marks*]

Jawapan / *Answer*:

(a) $g(x) = \dfrac{6}{(1-2x)^3} = 6(1-2x)^{-3}$

$g'(x) = -18(1-2x)^{-4}$

Maka/*Hence* $p = -18$ dan/*and* $q = -4$

$p + q = -22$

(b) $\dfrac{1}{m}\int g'(x)dx = \dfrac{-2}{81}$

$\dfrac{1}{m}g(x) = \dfrac{-2}{81}$

$\dfrac{6}{m(1-2x)^3} = \dfrac{-2}{81}$

Apabila/*When* $x = 2$, $\dfrac{6}{m(1-2(2))^3} = \dfrac{-2}{81}$

$\dfrac{6}{m(-27)} = \dfrac{-2}{81}$

$m = 9$

5. Diberi bahawa $\int_0^2 f(x)dx = \int_2^3 f(x)dx = 7$, nilaikan
Given that $\int_0^2 f(x)dx = \int_2^3 f(x)dx = 7$, *evaluate* SP 3.3.1

(a) $\int_0^3 f(x)dx$ [2 *markah* / 2 *marks*]

(b) $\int_{-2}^0 f(x)dx$ jika/*if* $f(x) = f(-x)$. [3 *markah* / 3 *marks*]

Jawapan / *Answer*:

(a) $\int_0^3 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$

$= 14$

(b) $\int_{-2}^0 f(x)\,dx = F(0) - F(-2)$

$\int_0^2 f(x)\,dx = F(2) - F(0)$

$= 7$

$F(2) = F(-2)$

$\int_{-2}^0 f(x)\,dx = -7$

6. Diberi $y = (x+3)(\sqrt{2x-3})$. Tunjukkan $\dfrac{dy}{dx} = \dfrac{3x}{\sqrt{2x-3}}$.
Seterusnya, cari nilai $\int_2^6 \dfrac{6x}{\sqrt{2x-3}}\,dx$. SP 3.3.1

Given $y = (x+3)(\sqrt{2x-3})$. *Show that* $\dfrac{dy}{dx} = \dfrac{3x}{\sqrt{2x-3}}$. *Then, find the value of* $\int_2^6 \dfrac{6x}{\sqrt{2x-3}}\,dx$.

[5 markah / 5 *marks*]

Jawapan / *Answer*:

$y = (x+3)(2x-3)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{(x+3)(2)}{2\sqrt{2x-3}} + \dfrac{\sqrt{2x-3}}{1}$

$= \dfrac{x+3+2x-3}{\sqrt{2x-3}}$

$= \dfrac{3x}{\sqrt{2x-3}}$

$\int \dfrac{3x}{\sqrt{2x-3}}dx = (x+3)\sqrt{(2x-3)}$

$\int_2^6 \dfrac{6x}{\sqrt{2x-3}}dx = [2(x+3)\sqrt{2x-3}]_2^6$

$= 2(9)(3) - 2(5)$

$= 44$

7. Diberi $\dfrac{d^2y}{dx^2} = -6x$ dan kecerunan lengkung itu pada $x = 2$ ialah -12 dan $y = 4$ apabila x mengambil nilai ini. SP 3.2.1

Given $\dfrac{d^2y}{dx^2} = -6x$ *and the gradient of the curve at* $x = 2$ *is* -12 *and* $y = 4$ *when* x *takes this value.*

(a) Ungkapkan y dalam sebutan x.
Express y *in terms of* x.
[3 markah / 3 *marks*]

(b) Cari nilai y *apabila* $x = 3$.
Find the value of y *when* $x = 3$.
[2 markah / 2 *marks*]

Jawapan / *Answer*:

(a) $\dfrac{d^2y}{dx^2} = -6x$

$\dfrac{dy}{dx} = -3x^2 + c$

Apabila/ *When* $x = 2$, $\dfrac{dy}{dx} = -12 + c = -12$

$\therefore c = 0$

$\dfrac{dy}{dx} = -3x^2$

$y = -x^3 + c$

Pada/*At* $(2, 4)$, $4 = -8 + c$

$c = 12$

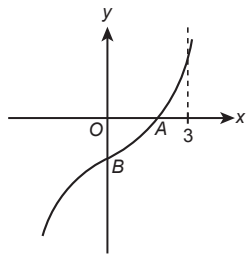
$\therefore y = -x^3 + 12$

(b) Apabila/*When* $x = 3$ $y = -27 + 12$

$y = -15$

KERTAS 2

1. $y = -x^2 - 8x - 15$ untuk ditukar kepada $y = x^3 - 8$.
$y = -x^2 - 8x - 15$ to be changed to $y = x^3 - 8$. SP 3.4.2



Kirakan/*Calculate*

(a) koordinat A dan B
the coordinates of A and B [4 *markah* / 4 *marks*]

(b) luas dibatasi oleh paksi-x dan paksi-y dan lengkung $y = x^3 - 8$.
the area enclosed by the x-axis and y-axis and the curve $y = x^3 - 8$, [3 *markah* / 3 *marks*]

(c) isipadu janaan dalam sebutan π apabila rantau dibatasi oleh lengkung, paksi-x dan $x = 3$ dikisarkan melalui 180° pada paksi-x.
the volume generated, in terms of π, when the region bounded by the curve, the x-axis and $x = 3$ is revolved 180° about the x-axis [3 *markah* / 3 *marks*]

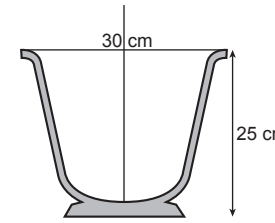
Jawapan / *Answer*:

(a) Apabila /*When* $y = 0,\ x^3 = 8,\ x = 2$
$\therefore A(2, 0)$
Apabila /*When* $x = 0,\ y = -8,\ B(0, -8)$

(b) Luas dibatasi $= \left|\int_0^2 y^2 dx\right| = \left|\int_0^2 (x^3 - 8)dx\right|$
The area enclosed
$= \left|\left[\frac{x^4}{4} - 8x\right]_0^2\right|$
$= |(4 - 16) - 0|$
$= |-12|$
$= 12 \text{ unit}^2$

(c) Isipadu janaan
The volume generated
$= \frac{\pi}{2}\int y^2\,dx = \frac{\pi}{2}\int_2^3 (x^3 - 8)^2 dx$
$= \frac{\pi}{2}\int_2^3 (x^6 - 16x^3 + 64)dx$
$= \frac{\pi}{2}\left[\frac{x^7}{7} - 4x^4 + 64x\right]_2^3$
$= 49\frac{1}{14}\pi \text{ unit}^3$

2. Rajah menunjukkan pandangan sisi permukaaan dalam bagi sebuah pasu bunga yang boleh diwakili oleh persamaan $y = px^2$. SP 3.3.4
The diagram shows the surface side view of a flower pot which can be represented by the equation $y = px^2$.



(a) Tunjukkan bahawa $a = \frac{1}{9}$.
Show that $a = \frac{1}{9}$.
[3 markah / 3 *marks*]

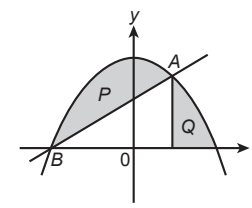
(b) Tentukan isi padu tanah, dalam cm³, yang diperlukan bagi mengisi pasu itu dengan kedalaman 20 cm.
Determine the volume of the ground, in cm³, that is needed to fill up the pot until the height is 20 cm.
[4 markah / 4 *marks*]

Jawapan / *Answer*:

(a) $y = ax^2$
Apabila/*When* $x = 15$ cm
$y = 25$ cm
$\therefore 25 = a(15)^2$
$a = \frac{1}{9}$

(b) Isi padu tanah
The volume of the ground
$= \pi\int_0^{20} x^2 dy$
$= \pi\int_0^{20} 9y\,dy$
$= \pi\left[\frac{9}{2}y^2\right]_0^{20}$
$= 5654.87 \text{ cm}^3$

3. Rajah menunjukkan garis lurus $y = x + 4$ menyilang lengkung $y = -\frac{1}{2}x^2 + 8$ pada A dan B. SP 3.4.2
The diagram shows a line $y = x + 4$ intersects the curve $y = -\frac{1}{2}x^2 + 8$ at A and B.



(a) Cari koordinat A dan B.
Find the coordinates of A and B.
[3 *markah* / 3 *marks*]

(b) Hitung luas berlorek P yang dibatasi oleh lengkung dan garis lurus.
Calculate the shaded area P which is bounded by the curve and the line.
[3 *markah* / 3 *marks*]

(c) Hitung isi padu janaan apabila luas berlorek Q yang dibatasi oleh lengkung, paksi-x dan garis mencancang yang melalui titik A diputarkan 360° pada paksi-x.

Calculate the volume generated when the shaded area Q which is bounded by the curve, the x-axis and the vertical line through A is rotated 360° about the x-axis.

[4 *markah* / 4 *marks*]

Jawapan / *Answer*:

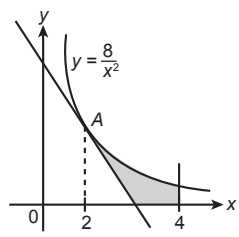
(a)
$$\frac{1}{2}x^2 + x - 4 = 0$$
$$x^2 + 2x - 8 = 0$$
$$(x-2)(x+4) = 0$$
$$\therefore x = 2\ ;\ -4$$
$$y = 6\ ;\ 0$$
$$\therefore A(2, 6)\quad B(-4, 0)$$

(b) Luas berlorek P = *The shaded area P*
$$= \int_{-4}^{2}\left[8 - \frac{1}{2}x^2\right]dx - \frac{1}{2}(6)(6)$$
$$= \left[8x - \frac{1}{6}x^3\right]_{-4}^{2} - 18$$
$$= \left[\left(16 - \frac{4}{3}\right) - \left(-32 + \frac{32}{3}\right)\right]$$
$$= 36 - 18$$
$$= 18 \text{ unit}^2$$

(c) Isi padu janaan = *The volume generated*
$$= \pi\int_{2}^{4} y^2 dx$$
$$= \pi\int_{2}^{4}\left(8 - \frac{1}{2}x^2\right)^2 dx$$
$$= \pi\int_{2}^{4}\left(64 - 8x^2 + \frac{1}{4}x^4\right) dx$$
$$= \pi\left[64x - \frac{8}{3}x^3 + \frac{1}{20}x^5\right]_{2}^{4}$$
$$= 28\frac{4}{15} \text{ unit}^3$$

4. Rajah menunjukkan lengkung $y = \frac{8}{x^2}$ dan garis lurus $y = mx + c$ adalah tangen kepada lengkung pada A dengan $x = 2$. SP 3.4.2

The diagram shows a curve $y = \frac{8}{x^2}$ and the line $y = mx + c$ is a tangent to the curve at A with $x = 2$.



(a) Cari nilai m dan nilai c

Find the value of m and of c. [3 markah / 3 *marks*]

(b) Cari luas yang dibatasi oleh lengkung, paksi-x, tangen di A *dan* $x = 4$.

Find the area bounded by the curve, the x-axis, the tangent at A and x = 4.

[3 markah / 3 *marks*]

(c) Diberi bahawa isi padu janaan apabila rantau dibatasi oleh lengkung, paksi-x, garis lurus $x = 2$ dan $x = k$ diputarkan melalui 360° pada paksi-x *ialah* $\frac{7\pi}{3}$ unit³. Cari nilai k *dengan keadaan* $k > 2$.

Given that the volume generated when the area bounded by the curve, the x-axis, the line x = 2 and x = k is rotated through 360° about the x-axis is $\frac{7\pi}{3}$ unit³. Find the value of k, where k > 2.

[4 markah / 4 *marks*]

Jawapan / *Answer*:

(a)
$$y = \frac{8}{x^2} = 8x^{-2}$$
$$\frac{dy}{dx} = -16x^{-3}$$
$$= \frac{-16}{x^3}$$
$$x = 2,\ \frac{dy}{dx} = -\frac{16}{8} = -2$$
$$\therefore y = -2x + c$$
$$(2, 2)\ ;\ 2 = -2(2) + c\ ;\ c = 6$$
$$y = -2x + 6$$
$$\therefore m = -2\ ;\ c = 6$$

(b) Luas yang dibatasi oleh lengkung, paksi-x, tangen di A dan $x = 4$

The area bounded by the curve, the x-axis, the tangent at A and x = 4
$$= \int_{2}^{4} y\,dx - \frac{1}{2}(2)(1)$$
$$= \int_{2}^{4} 8x^{-2}dx - 1$$
$$= \left[\frac{-8}{x}\right]_{2}^{4} - 1$$
$$= [-2 - (-4)] - 1$$
$$= 1 \text{ unit}^2$$

(c) Isi padu janaan $= \pi\int_{2}^{k} y^2 dx = \frac{7\pi}{3}$

The volume generated
$$\int_{2}^{k}[64x^{-4}]dx = \frac{7}{3}$$
$$\left[\frac{64}{-3x^3}\right]_{2}^{k} = \frac{7}{3}$$
$$\frac{64}{-3k^3} - \frac{\cancel{64}^{8}}{-3(\cancel{8})} = \frac{7}{3}$$
$$\frac{64}{-\cancel{3}k^3} = \frac{7}{3} - \frac{8}{3}$$
$$= \frac{-1}{3}$$
$$64 = k^3$$
$$k = 4$$

Aplikasi KBAT

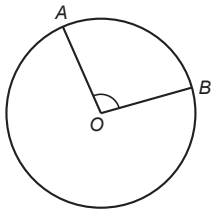
POT
Enrolment key ♥
KUA$aim+5

KUKUH KEMAHIRAN

BAB 1 Sukatan Membulat
Circular Measure

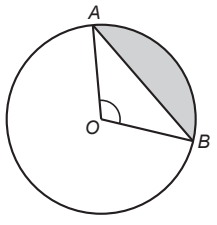
1. Cari nilai *x* bagi setiap sektor yang berikut di dalam sebuah bulatan.
Find the value of x for each of the following sectors of a circle.

	Jejari bagi sektor *Radius of sector* **(cm)**	**Sudut tercangkum di pusat bulatan dalam radian** *The subtended angle at the centre of a circle in radians*	**Panjang lengkok *AB*** *Arc length AB* **(cm)**
(a)	5.45	2.32	x
(b)	12.13	x	15.46
(c)	x	$\frac{3\pi}{5}$	32.72
(d)	$x + 3.5$	$\frac{2.1\pi}{8}$	$x + 2.4$



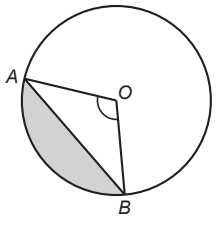
2. Cari nilai *x* bagi setiap sektor yang berikut di dalam sebuah bulatan.
Find the value of x for each of the following sectors of a circle.

	Jejari bagi sektor *Radius of sector* **(cm)**	**Sudut tercangkum di pusat bulatan dalam radian** *The subtended angle at the centre of a circle in radians*	**Perimeter tembereng berlorek** *Perimeter of the shaded segment* **(cm)**
(a)	2.8	2.1	x
(b)	4.3	3.25	x
(c)	20.2	$\frac{3\pi}{7}$	x
(d)	6.4	0.24π	x



3. Tentukan luas tembereng bagi setiap yang berikut.
Determine the area of segment for each of the following.

	Jejari *Radius* **(cm)**	**Sudut tercangkum di pusat bulatan dalam radian** *The subtended angle at the centre of a circle in radians*	**Luas tembereng berlorek** *Area of the shaded segment* **(cm²)**
(a)	7.2	1.2	x
(b)	14.3	0.6π	x
(c)	3.4	$\frac{5\pi}{9}$	x
(d)	51.5	1.4π	x



Jawapan Bab 1:

1. (a) 12.64 cm (b) 1.275 rad (c) 17.36 cm (d) 2.77
2. (a) 10.74 cm (b) 22.57 cm (c) 52.39 cm (d) 9.91 cm
3. (a) 6.95 cm² (b) 95.49 cm² (c) 9.08 cm² (d) 7093.82 cm²

BAB 2 Pembezaan
Differentiation

1. Cari $\frac{dy}{dx}$ dengan menggunakan prinsip pertama bagi setiap fungsi $y = f(x)$ yang berikut.
Find $\frac{dy}{dx}$ by using the first principles for each of the following functions of $y = f(x)$.

(a) $y = 2x - 5$

(b) $y = \frac{1}{2}x^2 + 3x$

(c) $y = 3 - 4x$

(d) $f(x) = 2(1 - 3x)$

2. Cari $f'(x)$ bagi setiap fungsi yang berikut.
Find $f'(x)$ for each of the following functions

(a) $f(x) = 5x + x^5$

(b) $f(x) = (x + 2)(5 - x^2)$

(c) $f(x) = \frac{5}{x^2} - \sqrt{x}$

(d) $f(x) = \frac{5}{\sqrt{1 - x}}$

3. Cari persamaan tangen dan normal kepada lengkung pada titik yang diberi seperti berikut.
Find the equation of the tangent and normal to the following curves at the given points.

(a) $y = 2x^3$ pada/*at* $x = 1$

(b) $y = \frac{1}{x}$ pada/*at* $x = -2$

(c) $y = 2x^2 + 4x$ pada/*at* $x = 1$

(d) $y = 4 - x^2$ pada/*at* $x = -2$

4. Cari titik pusingan bagi lengkung dan tentukan sama ada titik pusingan tersebut ialah titik maksimum, titik minimum atau titik refleks.
Find the turning point of the curve and determine whether the turning point is a maximum, minimum or reflex point.

(a) $y = 3x^2 - 6x$

(b) $y = 5x - 5x^2$

(c) $y = -x^3$

Jawapan Bab 2:

1. (a) 2
(b) $x + 3$
(c) -4
(d) -6

2. (a) $f'(x) = 5 + 5x^4$
(b) $f'(x) = 5 - 3x^2 - 4x$
(c) $f'(x) = \frac{-10}{x^3} - \frac{1}{2\sqrt{x}}$
(d) $f'(x) = -\frac{5}{2(1-x)\sqrt{1-x}}$

3. (a) $y = 6x - 4$; $y = -\frac{1}{6}x + \frac{13}{6}$
(b) $y = -\frac{1}{4}x$; $y = 4x + \frac{15}{2}$
(c) $y = 8x - 3$; $y = -\frac{1}{8}x + \frac{41}{8}$
(d) $y = 4x + 8$; $y = -\frac{1}{4}x - \frac{1}{2}$

4. (a) (1, −3), Titik minimum/ *Minimum point*
(b) $\left(\frac{1}{2}, \frac{5}{4}\right)$, Titik maksimum/ *Maximum point*
(c) (0, 0), Titik refleks/*Reflex point*

BAB 3 Pengamiran
Integration

1. Cari kamiran tak tentu terhadap x untuk yang berikut.
Find the indefinite integral with respect to x of the following.

(a) $\frac{1}{2}$

(b) $8x - 1$

(c) $\frac{2}{3}(x^2 - 6x + 3)$

(d) $x + \frac{1}{x^2}$

2. Cari / *Find*

(a) $\int \frac{x}{(2x)^3}dx$

(b) $\int \frac{x + 3}{x^4}dx$

(c) $\int \frac{x - 4x^2}{(1 - 4x)}dx$

(d) $\int \frac{x^2 - 5x - 6}{x - 6}dx$

3. Cari nilai yang berikut.
Find the value for the following.

(a) $\int_{-2}^{2}(2 + 5x)dx$

(b) $\int_{-1}^{1}\left(\frac{1}{2}x + 4x^{-2}\right)dx$

(c) $\int_{0}^{4}\left(\frac{2}{3}x + 3x^{-4}\right)dx$

(d) $\int_{0}^{5}\left(\frac{(x - 5)^3}{2}\right)dx$

Jawapan Bab 3:

1. (a) $\frac{1}{2}x + c$
(b) $4x^2 - x + c$
(c) $\frac{2}{9}x^3 - 2x^2 + 2x + c$
(d) $\frac{x^2}{2} - \frac{1}{x} + c$

2. (a) $-\frac{1}{8x} + c$
(b) $-\frac{1}{2x^2} - \frac{1}{x^3} + c$
(c) $\frac{1}{2}x^2 + c$
(d) $\frac{x^2}{2} + x + c$

3. (a) 8
(b) −8
(c) $\frac{61}{192}$
(d) $78\frac{1}{8}$

Pilih Atur dan Gabungan
Permutation and Combination

1. Nilaikan yang berikut tanpa kalkulator.
Evaluate the following without using a calculator.
(a) ${}^{12}P_6$
(b) ${}^{20}P_3$
(c) ${}^{n}P_1$

2. Nilaikan yang berikut tanpa kalkulator.
Evaluate the following without using a calculator.
(a) ${}^{14}C_9$
(b) ${}^{n}C_2$
(c) ${}^{2n}C_1$

3. Cari bilangan cara menyusun yang berikut dalam bulatan.
Find the number of ways to arrange the following in a circle.
(a) 12 pen/*pens*
(b) 5 pengawas/*prefects*
(c) 10 biji mutiara untuk menghasilkan seutas rantai.
10 pearls to make a chain.

4. Cari bilangan susunan pada satu garis lurus yang berikut.
Find the number of ways to arrange the following in a line.
(a) 4 pen secaman, 3 pensel secaman dan 6 batang pembaris secaman.
4 identical pens, 3 identical pencils and 6 identical rulers.
(b) 3 topi biru secaman dan 5 baju merah secaman.
3 identical caps and 5 identical red shirts.
(c) 6 cawan serupa, 7 sudu secaman dan 3 piring secaman.
6 identical cups, 7 identical spoons and 3 identical plates.

5. NIlaikan yang berikut.
Evaluate the following.
(a) Memilih 5 batang pensel daripada 8 pensel yang berlainan.
Choose 5 pencils from 8 different pencils.
(b) Memilih 4 orang pengawas daripada 7 orang pengawas.
Choose 4 prefects from 7 prefects.
(c) Memilih 2 biji bola daripada 8 biji bola dari bakul.
Choose 2 balls out of 8 balls from the basket.

Jawapan Bab 4:

1. (a) 665 280
(b) 6 840
(c) n

2. (a) 2 002
(b) $\frac{n(n-1)}{2}$
(c) $2n$

3. (a) 11!
(b) 4!
(c) $\frac{9!}{2}$

4. (a) 60 060
(b) 56
(c) 960 960

5. (a) 56
(b) 35
(c) 28

Taburan Kebarangkalian
Probability Distribution

1. Sekeping duit syiling adil dilambungkan 7 kali. Cari kebarangkalian untuk mendapat
A coin is tossed 7 times. Find the probability that it shows
(a) gambar 3 kali.
the heads of 3 times.
(b) gambar lebih daripada 5 kali.
the heads of more than 5 times.
(c) gambar kurang daripada 5 kali.
the heads of less than 5 times.

2. Sebiji dadu adil dilambungkan sebanyak 5 kali. Cari kebarangkalian untuk mendapat
A dice is tossed 5 times. Find the probability that it shows
(a) nombor 2 tepat 2 kali.
number 2 exactly 2 times.
(b) nombor 2 sekurang-kurangnya 2 kali.
number 2 at least 2 times.
(c) nombor 2 selebih-lebihnya 2 kali.
number 2 at most 2 times.

3. Jika/*If* $X \sim B\left(8, \frac{1}{3}\right)$, cari/*find*
(a) $P(X = 3)$
(b) $P(X > 2)$
(c) $P(X < 7)$

4. Jika X ialah pemboleh ubah rawak selanjar yang bertaburan normal dengan min 10 dan varians 16, cari
If X is a continuous random variable with normal distribution of mean 10 and variance 16, find
(a) $P(X < 13)$
(b) $P(5 < X < 9)$
(c) $P(7.5 < X < 15)$
(d) $P(|X| > 15)$

5. Pemboleh ubah rawak selanjar X bertaburan normal dengan min 40 dan varians 4, cari nilai k jika
X is a continuous random variable with normal distribution of mean 40 and variance 4, find the value of k if
(a) $P(X > k) = 0.25$
(b) $P(X < k) = 0.6331$
(c) $P(X < k + 45) = 0.0018$
(d) $P(X > k - 5) = 0.05$

Jawapan Bab 5:

1. (a) 0.2734 (b) 0.0625 (c) 0.7734
2. (a) 0.1608 (b) 0.1962 (c) 0.9645
3. (a) 0.2731 (b) 0.5318 (c) 0.9974 (d) 0.2113
4. (a) 0.7734 (b) 0.2956 (c) 0.6284
5. (a) 41.35 (b) 40.68 (c) 0.82 (d) 50.15

1. Diberi sin $A = \frac{3}{5}$ dan kos $B = \frac{12}{13}$, dengan keadaan A dan B adalah sudut tirus, cari

Given sin $A = \frac{3}{5}$ and cos $B = \frac{12}{13}$, such that A and B are acute angles, find

(a) tan A
(b) sin $(A + B)$
(c) kos/*cos* $(B - A)$
(d) kos/*cos* $2B$
(e) tan $(A + B)$
(f) sin $(B - A)$
(g) sek/*sec* $(A + B)$
(h) kosek/*cosec* $(B - A)$

2. Padankan rumus dengan betul.
Match the correct formulae.

(a)	tan $(A + B)$ =
(b)	sin A sin B – kos A kos B = *sin A sin B – cos A cos B =*
(c)	kos A kos B + sin A sin B = *cos A cos B + sin A sin B =*
(d)	sin $(A + B)$ =
(e)	kos A kos B – sin A sin B = *cos A cos B – sin A sin B =*
(f)	kos^2A – sin^2 A = *cos^{2}A – sin^{2}A =*
(g)	tan $(A - B)$ =
(h)	sin $(A - B)$ =

sin A kos B + kos A sin B *sin A cos B + cos A sin B*
sin $(A - B)$
$\frac{\tan A + \tan B}{1 - \tan A \tan B}$
kos $(A + B)$ *cos (A + B)*
sin $(A + B)$
$\frac{\tan A - \tan B}{1 + \tan A \tan B}$
$\frac{\tan A - \tan B}{1 - \tan A \tan B}$
kos $(A - B)$ *cos (A – B)*
sin A kos B – kos A sin B *sin A cos B – cos A sin B*
$\frac{\tan A + \tan B}{1 + \tan A \tan B}$

Jawapan Bab 6:

1. (a) $\frac{3}{4}$

(b) $\frac{56}{65}$

(c) $\frac{63}{65}$

(d) $\frac{119}{169}$

(e) $\frac{56}{33}$

(f) $-\frac{16}{65}$

(g) $\frac{65}{33}$

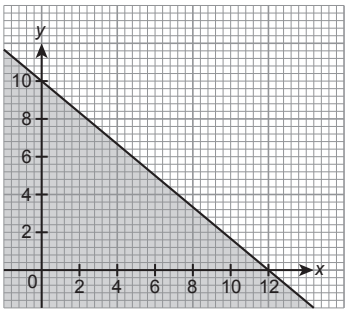
(h) $-\frac{65}{16}$

2.

(a)	tan $(A + B)$ =	sin A kos B + kos A sin B *sin A cos B + cos A sin B*
(b)	sin A sin B – kos A kos B = *sin A sin B – cos A cos B =*	sin $(A - B)$
(c)	kos A kos B + sin A sin B = *cos A cos B + sin A sin B =*	$\frac{\tan A + \tan B}{1 - \tan A \tan B}$
(d)	sin $(A + B)$ =	kos $(A + B)$ *cos (A + B)*
(e)	kos A kos B – sin A sin B = *cos A cos B – sin A sin B =*	sin $(A + B)$
(f)	kos^2A – sin^2 A = *cos^{2}A – sin^{2}A =*	$\frac{\tan A - \tan B}{1 + \tan A \tan B}$
(g)	tan $(A - B)$ =	$\frac{\tan A - \tan B}{1 - \tan A \tan B}$
(h)	sin $(A - B)$ =	kos $(A - B)$ *cos (A – B)*
		sin A kos B – kos A sin B *sin A cos B – cos A sin B*
		$\frac{\tan A + \tan B}{1 + \tan A \tan B}$

1. Terdapat *x* orang perempuan dan *y* orang lelaki mengambil bahagian dalam pertandingan menulis karangan. Diketahui bahawa nisbah bilangan perempuan kepada bilangan lelaki adalah tidak lebih daripada 4 : 5 dan bilangan lelaki melebihi bilangan perempuan sebanyak sekurang-kurangnya 3. Tulis satu set ketaksamaan yang mewakili situasi tersebut.
There are x girls and y boys taking part in the school essay writing competition. It is known that the ratio of the number of girls to boys is not more than 4 : 5 and the number of boys exceeds the number of girls by at least 3. Write down the set of inequalities that represent the situation

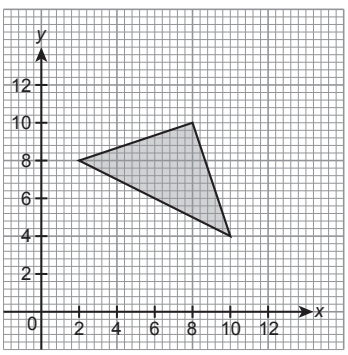
2. Graf menunjukkan jumlah perbelanjaan Wan apabila diberikan wang RM*W* untuk membeli *x* buku kerja dan *y* buku latihan. Harga setiap buku kerja dan buku latihan adalah masing-masing RM10 dan RM12.
The graph shows the amount of Wan's expenses when he is given RMW of money to buy x workbooks and y exercise books. The price of each workbook and exercise book is RM10 and RM12 respectively.



(a) Tulis satu ketaksamaan yang mentakrifkan rantau tersebut, selain daripada x ≥ 0 dan y ≥ 0. Cari nilai *W*.
Write an inequality that defines that region, other than x ≥ 0 and y ≥ 0. Find the value of W.

(b) Tulis dalam ayat yang mentakrifkan rantau berlorek yang diberi, selain daripada *x ≥ 0* dan *y ≥ 0*.
Write a sentence which defines the shaded region given, other than x ≥ 0 and y ≥ 0.

3. Rajah menunjukkan suatu rantau berlorek.
The diagram shows the shaded region.



Cari nilai minimum dan maksimum bagi fungsi objek $P = 10x - 4y$.
Find the minimum and maximum values for the objective function P = 10x – 4y.

Jawapan Bab 7:

1. (a) $\frac{x}{y} \leqslant \frac{4}{5}$ atau/*or* $5x \leqslant 4y$ atau/*or* $y - x \geqslant 3$

2. (a) $12y + 10x \leqslant 120$; $W = 120$
(b) Jumlah perbelanjaan Wan tidak melebihi RM120.
Wan's total expenses does not exceed RM120.

3. Nilai minimum/*Minimum value* = –12
Nilai maksimum/*Maximum value* = 84

Kinematik Gerakan Linear
Kinematics of Linear Motion

1. Sesaran suatu zarah dari titik tetap *O* pada masa *t* saat diberi oleh *s* meter. Ungkapkan halaju *v* dan pecutan *a,* dalam sebutan *t,* bagi yang berikut.
The displacement of a particle from a given point O at t seconds is given by s metres. Express the velocity v and acceleration a, in terms of t, for the following.

(a) $s = t^3 - 12t^2 + 36t$

(b) $s = -\frac{1}{3}t^3 + 4t^2$

(c) $s = (t - 4)(t^2 - 2)$

(d) $s = \sqrt{t + 2}$

2. Pecutan, a m s^{-2}, suatu zarah yang bergerak pada satu garis lurus diberi seperti berikut. Cari ungkapan bagi halaju, v m s^{-1}, dan sesaran, s m, bagi setiap yang berikut, dengan syarat-syarat yang diberi.
The acceleration, a m s^{-2}, for a particle that is moving on a straight line is given as follows. Find the expression for the velocity, v m s^{-1}, and the displacement, s m, for each of the following with the given conditions.

(a) $a = 2t$, apabila/*when* $t = 1$ s, $s = 1$ m dan/*and* $v = -2$ m s^{-1}

(b) $a = 2 - t$, apabila/*when* $t = 0$ s, $s = -2$ m dan/*and* $v = 5$ m s^{-1}

(c) $a = \frac{1}{2} - 4t$, apabila/*when* $t = 2$ s, $s = -4$ m dan/*and* $v = 0$ m s^{-1}

3. Halaju, v m s^{-1}, suatu zarah yang bergerak pada satu garis lurus diberi seperti berikut. Cari ungkapan bagi pecutan, a m s^{-2}, dan sesaran, s m, bagi setiap yang berikut dengan syarat-syarat yang diberi.
The velocity, v m s^{-1}, for a particle that is moving on a straight line is given as follows. Find the expression for the acceleration, a m s^{-2}, and the displacement, s m, for each of the following with the given conditions.

(a) $v = (t - 4)(2t + 1)$ apabila/*when* $t = 0$, $s = 0$ m

(b) $v = t^2(2t - 1)$ apabila/*when* $t = 2$, $s = -1$ m

(c) $v = -(t - 6)^2 + 2$ apabila/*when* $t = 0$, s = 0 m

Jawapan Bab 8:

1. (a) $v = 3t^2 - 24t + 36$; $a = 6t - 24$
(b) $v = -t^2 + 8t$; $a = -2t + 8$
(c) $v = 3t^2 - 8t - 2$; $a = 6t - 8$
(d) $v = \frac{1}{2\sqrt{t-2}}$; $a = \frac{1}{4(t+2)\sqrt{t+2}}$

2. (a) $v = t^2 - 3$; $s = \frac{t^3}{3} - 3t = \frac{11}{3}$
(b) $v = 2t - \frac{1}{2}t^2 + 5$; $s = t^2 - \frac{1}{6}t^3 + 5t - 2$
(c) $v = \frac{1}{2}t - 2t^2 + 7$; $s = \frac{1}{4}t^2 - \frac{2}{3}t^3 + 7t - \frac{41}{3}$

3. (a) $a = 4t - 7$; $s = \frac{2}{3}t^3 - \frac{7}{2}t^2 - 4t$
(b) $a = 6t^2 - 2t$; $s = \frac{1}{2}t^4 - \frac{1}{3}t^3 - \frac{19}{3}$
(c) $a = -2(t - 6)$; $s = \frac{-(t-6)^3}{3} + 2t - 72$