

TARGET

EDISI GURU

PBD

MODUL PENTAKSIRAN BILIK DARJAH

TINGKATAN 5

KSSM

MATEMATIK TAMBAHAN ADDITIONAL MATHEMATICS



Melancarkan
Pentaksiran Bilik
Darjah (PBD)



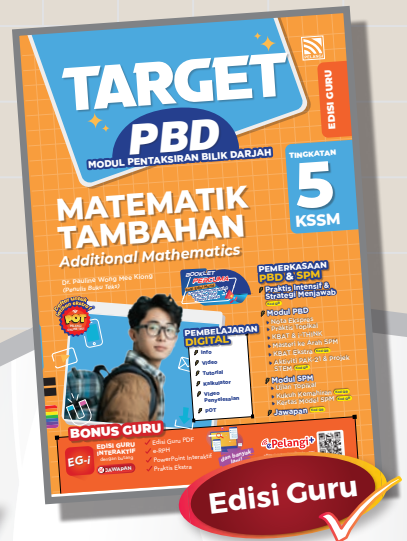
Memantapkan
Pentaksiran sumatif
& SPM



Menyokong
Pembelajaran dan
Pemudaharaan
(PdPc) Mesra Digital



Meningkatkan
Tahap Penguasaan
Murid



Edisi Guru

PAKEJ PERCUMA UNTUK KEMUDAHAN GURU

EDISI GURU

VERSI CETAK

PEMERKASAAN PBD & SPM

- ⚡ Modul PBD
- ⚡ Modul SPM
- ⚡ Jawapan

PEMBELAJARAN DIGITAL

- ⚡ Pelbagai bahan sokongan pembelajaran dalam talian

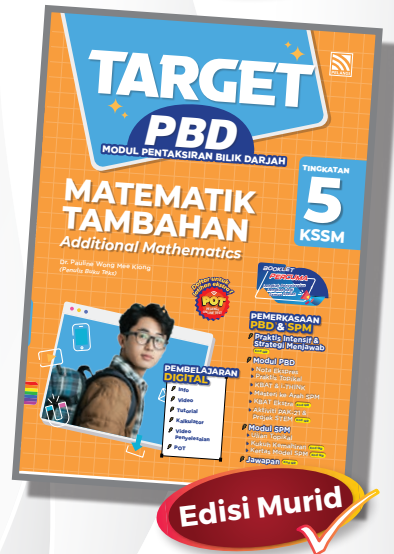
RESOS DIGITAL GURU



Pelbagai bahan digital sokongan PdPc yang disediakan khas untuk guru di platform ePelangi+



BAHAN
SOKONGAN
PdPc
EKSTRA!



Edisi Murid



Kandungan

Kandungan mengemukakan bahagian-bahagian buku berserta rujukan bahan-bahan digital sokongan dalam buku.

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Rekod Pentaksiran Murid

Jadual untuk catatan prestasi Tahap Penguasaan murid.

REKOD PENTAKSIRAN MURID					
MATEMATIK TAMBAHAN (Tingkatan 5)					
Nama: _____					
BAB	TAHAP PENGUASAAN	STANDARD PRESTASI		PENCAPAIAN	
		TAFSIRAN	HALAMAN	(%) MENGEKSAI	(%) BELUM MENGEKSAI
BIDANG PEMBELAJARAN: GEOMETRI					
1	Sukatan Membulat	TP1	Mengaplikasikan pengetahuan asas tentang sukatan membulat.	1 – 2	
		TP2	Mempamerkan kefahaman tentang sukatan membulat.	3	
		TP3	Mengaplikasikan kefahaman tentang sukatan membulat untuk melaksanakan tugasan mudah.	3, 6	
		TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah rutin yang kompleks.	4, 7 – 8	
		TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah rutin yang kompleks.	5, 9 – 12	
		TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang sukatan membulat dalam konteks penyelesaian masalah bukan rutin secara kreatif.	12	
BIDANG PEMBELAJARAN: KALKULUS					
2	Pembezaan	TP1	Mengaplikasikan pengetahuan asas tentang pembezaan.	15	
		TP2	Mempamerkan kefahaman tentang pembezaan.	15 – 16	
		TP3	Mengaplikasikan kefahaman tentang pembezaan untuk melaksanakan tugasan mudah.	15 – 17, 19 – 20	
		TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah rutin yang mudah.	17 – 19, 20 – 21, 23, 25	
		TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah rutin yang kompleks.	27 – 28	
		TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pembezaan dalam konteks penyelesaian masalah bukan rutin secara kreatif.	22, 24, 26 – 27	
BIDANG PEMBELAJARAN: KALKULUS					
3	Pengamiran	TP1	Mengaplikasikan pengetahuan asas tentang pengamiran.	31	
		TP2	Mempamerkan kefahaman tentang pengamiran.	32	
		TP3	Mengaplikasikan kefahaman tentang pengamiran untuk melaksanakan tugasan mudah.	33 – 34	



Nota Ekspres

Nota ringkas yang mudah diikuti oleh murid dan mencakupi setiap unit.

BAB 1 **MODUL PBD**
Sukatan Membulat
Circular Measure

1.1 Radian
Radian

NOTA EKSPRES

1. Sudut yang dicangkup pada pusat bulatan oleh lengkok yang sama panjang dengan jejariya ditakrifkan sebagai 1 radian.
1 radian is the angle subtended at the centre of the circle by the arc length which is the same length as the radius.

2. Sudut dicangkup pada pusat bulatan oleh satu bukitan ialah 2π radian yang setara dengan 360°. The angle subtended at the centre of a circle by the circle is 2π radians which is equivalent to 360°.

3. Hubungan antara sudut diukur dalam darjah dan radian bagi suatu sektor yang berikut:
The relation between angle measured in degree and radian for a sector is as follows:

$$\frac{\theta^\circ}{360^\circ} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} \text{ atau setaranya } 1 \text{ or equivalently } \frac{\theta^\circ}{180^\circ} = \frac{\theta \text{ rad}}{\pi \text{ rad}}$$

TUTORIAL

1. Tukarkan setiap yang berikut kepada darjah dan minit. **12A**
Convert each of the following into degrees and minutes.
Guna/Use $\pi = 3.142$.

Contoh

(a) 5.15 rad
 $1 \text{ rad} = 180^\circ$
 $5.15 \text{ rad} = 5.15 \times \frac{180^\circ}{\pi}$
 $= 295^\circ 2'$

(b) $\frac{3}{7}\pi$ rad
 $1 \text{ rad} = 180^\circ$
 $\frac{3}{7}\pi \text{ rad} = \frac{3}{7} \times 180^\circ$
 $= 77^\circ 9'$

2. Tukarkan setiap yang berikut kepada radian. **12B**
Convert each of the following into radians.
Guna/Use $\pi = 3.142$.

Contoh

(a) 134° 24'
 $180^\circ = \pi \text{ rad}$
 $134^\circ 24' = 134^\circ 24' \times \frac{\pi}{180^\circ}$
 $= 134^\circ 24' \times \frac{3.142}{180^\circ}$
 $= 2.346 \text{ rad}$

(b) 294° 6'
 $180^\circ = \pi \text{ rad}$
 $294^\circ 6' = 294^\circ 6' \times \frac{\pi}{180^\circ}$
 $= 294^\circ 6' \times \frac{3.142}{180^\circ}$
 $= 5.134 \text{ rad}$





BAB 4 MODUL PBD
SOALAN PEMBELAJARAN

Pilih Atur dan Gabungan

Permutation and Combination

4.1 Pilih Atur Permutation

NOTA EKSPRES

- Jika suatu peristiwa boleh berlaku dalam m cara dan suatu peristiwa kedua boleh berlaku dalam n cara, maka kedua-dua peristiwa boleh berlaku dalam $m \times n$ cara.
If an event can happen in m ways and another event can happen in n ways, then the two events can happen successively in $m \times n$ ways.
- Jika n objek yang berbeza disusun dalam satu baris, bilangan pilih atur ialah P_n , atau $n!$.
If n different objects are arranged in a row, the number of permutations is P_n , or $n!$.
- " P_n " boleh ditulis dalam bentuk " $n!$ ", dan boleh ditulis dalam bentuk:
(a) $n!$
(b) $n \times (n-1) \times \dots \times 3 \times 2 \times 1$
- Bilangan pilih atur menyusun 0 objek ialah $0! = 1$.
The number of ways to arrange objects is $0! = 1$.
- Contoh-contoh pilih atur.
Examples of permutations:
(a) $P_3 = 3!$ (b) $P_4 = 4!$ (c) $P_5 = 5!$
(d) $n! = 1 \times 2 \times \dots \times n$ (e) $P_n = n!$ (f) $P_n = n!$
(g) $n! = 1 \times 2 \times \dots \times 2 \times 2 \times \dots \times 2$

1. Selesaikan masalah berikut. Solve the following problems.

Contoh
Tom ingin memilih 1 batang pen dan 1 batang pembaris dari suatu dalgang yang mengandungi 8 batang pen dan 7 batang pembaris. Berapa cara berlainan Tom boleh melakukannya?
Tom wants to choose a pen and a ruler from a tray which contains 8 pens and 7 rulers. In how many ways can this be done?

(a) Terdapat 4 jenis seluar dan 5 jenis baju yang dijual di sebuah kedai. Cari bilangan cara Shirley boleh membeli sepasang seluar dan sepasang baju.
There are 4 types of trousers and 5 types of blouses being sold at a shop. Find the number of ways Shirley can buy a pair of trousers and a blouse.

Bilangan cara = $4 \times 5 = 20$
Number of ways

Tom terdapat 8 cara untuk memilih sebatang pen dan 7 cara untuk memilih sebatang pembaris dari dalgang itu.
Mengikut petua pendaraban, jumlah cara ialah $8 \times 7 = 56$.
Tom has 8 ways to choose a pen and 7 ways to choose a ruler from the tray.
By using multiplication rule, the total number of ways is $8 \times 7 = 56$.

- 1 Praktis topikal yang menilai kesemua Tahap Penguasaan (TPI-6) yang tercakup dalam DSKP.
- 2 Soalan yang mematuhi Standard Kandungan (SK) dan Standard Pembelajaran (SP) serta menepati kandungan dalam buku teks.
- 3 Bahan pembelajaran digital seperti Info, Video, Video Tutorial, Video Penyelesaian, Kalkulator dan KBAT Ekstra menyokong pembelajaran yang kondusif.



- 4 Soalan Kemahiran Berfikir Aras Tinggi (KBAT) untuk mencabar pemikiran murid.
- 5 Aktiviti seperti Projek STEM dan PAK-21 disertakan untuk menyempurnakan PdPc.
- 6 **Masteri ke Arah SPM** memberikan panduan langkah demi langkah untuk menjawab soalan berpiawai SPM. Soalan yang serupa disediakan untuk membolehkan murid berlatih secara efektif.



Matematik Tambahan Tingkatan 5 Bab 1 Soalan Membantu

1.4 Aplikasi Sukatan Membulat

Application of Circular Measures

13. Selesaikan masalah yang berikut. Solve the following problems.

Contoh
Rajah menunjukkan satu corak yang dibina daripada dua sektor serupa menyentuh satu sama lain di O dan dengan pusat masing-masing A dan B . Diberi panjang perentas PO ialah 16 cm dan $AB = 24$ cm. Cari
The diagram shows a pattern made up of two similar sectors with centres A and B respectively and touch each other at O . Given that the chord PO is 16 cm and $AB = 24$ cm. Find
(i) $\angle POQ$ dalam radian.
(ii) $\angle PAQ$ dalam radian.
(iii) perimeter rantau berlekeh, the perimeter of the shaded region,
(iv) luas rantau berlekeh.

PENYELESAIAN
(i) $\sin \theta = \frac{8}{12} = 0.6667$
 $\theta = 41.81^\circ = 0.73 \text{ rad}$
 $\angle POQ = 1.46 \text{ rad}$
(ii) Perimeter rantau berlekeh
Perimeter of the shaded region
= $2(16 + 12(1.46))$
= 67.04 cm
(iii) Luas rantau berlekeh
Area of shaded region
= $\frac{1}{2}(12)^2(1.46) - \frac{1}{2}(12)^2(\sin 83.62^\circ)$
= 67.13 cm^2

PROJEK STEM
Aplikasi sukatan membulat
Aplikasi

PAK-21
Kemahiran berfikir aras tinggi
Menganalisis

KBAT EKSTRA

14. Rajah menunjukkan sekeping tanah yang berbentuk trapezium $ABCD$ kepunyaan Henri, dengan panjang $AD = BC$ dan $DC = 2AB$. Henri ingin megar kawasan itu dengan pagar 340 m. The diagram shows a piece of land in the shape of trapezium $ABCD$ owned by Henri, with the lengths $AD = BC$ and $DC = 2AB$. Henri wants to fence his land with 340 m of fencing.
(i) Cari $\angle ADC$ dalam radian.
(ii) Sektor ADE berpusat D dan sektor ECF berpusat C akan digunakan untuk menanam sayur dan bahagian yang tertinggal diliputi dengan rumput. Cari luas yang diliputi dengan rumput.
The sector ADE with centre D and sector ECF with centre C will be used to plant vegetables, and the remaining will be covered with grass. Find the area covered by grass.

(i) Panjang AD / Length of AD
 $= \frac{340 - 60 - 120}{2} = 80 \text{ m}$
 $\cos \angle ADC = \frac{30}{80}$
 $\angle ADC = 67.98^\circ = 1.1866 \text{ rad}$
 $67.98^\circ - 67.98^\circ \times 1.1866 \text{ rad}$
 $\angle ADC = 1.1866 \text{ rad}$
(ii) Luas trapezium $ABCD$ / Area of trapezium $ABCD$
 $= \frac{1}{2}(60 + 120)(80 \sin 67.98^\circ)$
 $= 6674.78 \text{ m}^2$
Luas sektor ADE / Area of sector ADE
 $= \frac{1}{2}(80)(1.1866)$
 $= 3797.12 \text{ m}^2$
Luas sektor ECF / Area of sector ECF
 $= \frac{1}{2}(40)(1.1866)$
 $= 949.28 \text{ m}^2$
Luas diliputi rumput / Area covered by grass
 $= 6674.78 - 3797.12 - 949.28$
 $= 1928.38 \text{ m}^2$

6 MASTERI KE ARAH SPM

Walter telah mendaftar akaun media sosial yang baharu. Dia perlu menetapkan kata laluan dengan memilih 5 aksara daripada 14 pilihan aksara yang diberi dalam jadual di bawah.
Walter has registered a new social media account. He needs to set a password by choosing 5 characters from the given options of 14 characters in the table below.

Huruf besar Capital letters	Huruf kecil Small letters	Nombor Numbers	Simbol Symbols
A, S, K	a, s, k	1, 2, 5, 8, 9	%, *, @

Cari bilangan kata laluan berbeza dapat dibentuk jika
Find the number of different passwords that can be formed if

(a) dua aksara pertama ialah 'S' tanpa ulangan.
the first two characters are 'S' without repetition.
(b) dua aksara terakhir mesti mengandungi huruf besar diikuti oleh simbol dan susunan mempunyai huruf kecil di tengah. Ulangan tidak dibenarkan.
the last two characters must contain a capital letter followed by a symbol and the arrangement has small letters in the middle. Repeats are not allowed.

Faham
(a) dua aksara pertama ialah 'S' tanpa ulangan
the first two characters are 'S' without repetition.

Aksara yang dipilih	Aksara yang boleh dipilih	Aksara yang boleh dipilih
S	P	Other

(b) dua aksara terakhir mesti mengandungi huruf besar diikuti oleh simbol dan susunan mempunyai huruf kecil di tengah. Ulangan tidak dibenarkan.
the last two characters must contain a capital letter followed by a symbol and the arrangement has small letters in the middle. Repeats are not allowed.

Tutut
(a) 12 aksara masih tinggal. Perlu susun 3 daripadanya bilangan cara/huruf dan susunan
 $= P_{12} = 1320$

Aksara yang dipilih	Aksara yang boleh dipilih	Huruf besar Capital letters	Simbol Symbols
yg	yg	3 cara 3 ways	3 cara 3 ways

Maka bilangan cara
Thus, the number of ways
 $= 12 \times 12 \times 4 \times 3 \times 3$
 $= 3960$

CUBA DIRI

Harini telah mendaftar akaun media sosial yang baharu. Dia perlu menetapkan kata laluan dengan memilih 5 aksara daripada 14 pilihan aksara yang diberi dalam jadual di bawah.
Harini has registered a new social media account. He needs to set a password by choosing 5 characters from the 14 characters options listed in the table below.

Huruf besar Capital letters	Huruf kecil Small letters	Nombor Numbers	Simbol Symbols
M, P, T, U	t, s	1, 3, 7	%, *, @

Cari bilangan kata laluan berbeza dapat dibentuk jika
Find the number of different passwords that can be formed if

(a) dua aksara terakhir ialah 'M' tanpa ulangan.
the last two characters are 'M' without repetition.
(b) kata laluan mesti mengandungi hanya dua huruf besar dengan satu simbol di antaranya. Ulangan tidak dibenarkan.
the password must contain only two capital letters with a symbol between them. Repeats are not allowed.

$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$
 $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$
 $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$
 $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$
 $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
 $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
 $4! = 4 \times 3 \times 2 \times 1 = 24$
 $3! = 3 \times 2 \times 1 = 6$
 $2! = 2 \times 1 = 2$
 $1! = 1$
 $0! = 1$

E Modul SPM » Pentaksiran Sumatif

- 1 Ujian-ujian topikal dengan soalan-soalan berpiawai SPM.
- 2 Kukuh Kemahiran – soalan-soalan latihan asas bagi setiap bab **Kod QR**
- 3 Kertas Model SPM **Kod QR**
- 4 **Pembelajaran digital** melibatkan Pelangi Online Test (POT)



UJIAN	SKOP	HALAMAN
UJIAN 1	Solatan Mambuat	127
UJIAN 2	Pembezaan	134
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Kod QR icons for each section.

UJIAN 3

skema

1. Diberikan bahawa $\frac{d}{dx} \left[\frac{2x^2-3}{(1-x)^2} \right] = \frac{2}{(1-x)^3}$ cari

Jawapan / Answer:

2. Suatu lengkung melalui titik P(4, -1), mempunyai

Jawapan / Answer:

KERTAS 3

Bagian B

1. (a) Lalar graf $y = x^2 + 8x - 2$.
 (b) Tentukan, cari luas yang dibatasi oleh

Bagian A

1. Rajah di bawah menunjukkan suatu bulatan

F Jawapan

Jawapan keseluruhan buku **Kod QR** disediakan di halaman Kandungan.



G Praktis Intensif & Strategi Menjawab

Pelbagai bentuk soalan berserta strategi menjawab sebagai latihan sendiri untuk murid **Kod QR** disediakan di halaman Kandungan.



6.1.1	6.1.2	6.1.3	6.1.4	6.1.5	6.1.6	6.1.7	6.1.8	6.1.9	6.1.10	6.1.11	6.1.12	6.1.13	6.1.14	6.1.15	6.1.16	6.1.17	6.1.18	6.1.19	6.1.20	6.1.21	6.1.22	6.1.23	6.1.24	6.1.25	6.1.26	6.1.27	6.1.28	6.1.29	6.1.30	6.1.31	6.1.32	6.1.33	6.1.34	6.1.35	6.1.36	6.1.37	6.1.38	6.1.39	6.1.40	6.1.41	6.1.42	6.1.43	6.1.44	6.1.45	6.1.46	6.1.47	6.1.48	6.1.49	6.1.50	6.1.51	6.1.52	6.1.53	6.1.54	6.1.55	6.1.56	6.1.57	6.1.58	6.1.59	6.1.60	6.1.61	6.1.62	6.1.63	6.1.64	6.1.65	6.1.66	6.1.67	6.1.68	6.1.69	6.1.70	6.1.71	6.1.72	6.1.73	6.1.74	6.1.75	6.1.76	6.1.77	6.1.78	6.1.79	6.1.80	6.1.81	6.1.82	6.1.83	6.1.84	6.1.85	6.1.86	6.1.87	6.1.88	6.1.89	6.1.90	6.1.91	6.1.92	6.1.93	6.1.94	6.1.95	6.1.96	6.1.97	6.1.98	6.1.99	6.1.100
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JAWAPAN BAB 3

3. Pengamiran

1. (a) Pembedaan bagi $5x^2 + 3x$ ialah $10x + 3$.

(b) Pembedaan bagi $\frac{x}{1-x^2}$ ialah $\frac{1}{(1-x)^2}$.

(c) Pembedaan bagi $(2x-1)^2$ ialah $4(2x-1)$.

2. (a) $\int \frac{x^2-4}{x^2+4} dx = \int \frac{x^2+4-4}{x^2+4} dx = \int \frac{x^2+4}{x^2+4} dx - \int \frac{4}{x^2+4} dx = x - \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + c$

(b) $\int \frac{3x^2+2}{x^2+1} dx = \int \frac{3x^2+3-1}{x^2+1} dx = \int \frac{3(x^2+1)-1}{x^2+1} dx = 3 \int \frac{x^2+1}{x^2+1} dx - \int \frac{1}{x^2+1} dx = 3x - \tan^{-1} x + c$

(c) $\int \frac{2x^2+3x-4}{x^2-1} dx = \int \frac{2x^2+3x-4}{(x-1)(x+1)} dx = \int \frac{A}{x-1} + \frac{B}{x+1} dx = A \ln|x-1| + B \ln|x+1| + c$

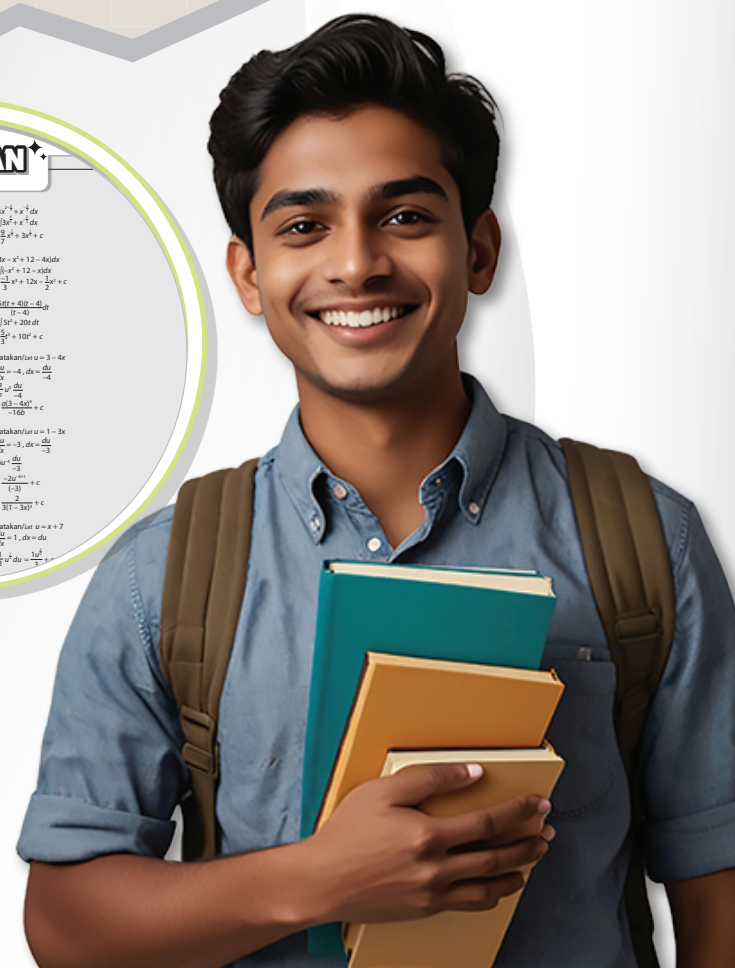
Praktis Intensif & Strategi Menjawab


Bab 1

1. Tentukan perimeter terbiting yang bertolak bagi bulatan tersebut O dengan dibariskan jejari oleh sudut POQ.

2. Diberi suatu luas sebuah bulatan bulatan O ialah 401 cm² dan panjang lengkok PQ ialah 20 cm.

3. Diberi suatu luas sebuah bulatan bulatan O ialah 401 cm² dan panjang lengkok PQ ialah 20 cm.



Di platform , guru yang menerima guna (*adoption*) siri Target PBD KSSM diberi akses kepada EG-i dan bahan sokongan ekstra PdPc untuk tempoh satu tahun:

1 Apakah itu ?



EG-i merupakan versi digital dan interaktif Edisi Guru Target PBD secara dalam talian. Versi ini akan dapat mengoptimumkan penggunaan teknologi dalam pengajaran, memaksimumkan kesan PdPc, dan membangunkan suasana pembelajaran yang menyeronokkan serta responsif dalam kalangan murid.



PANDUAN PENGGUNAAN


Halaman Contoh

Alat sokongan lain:

-  Pen
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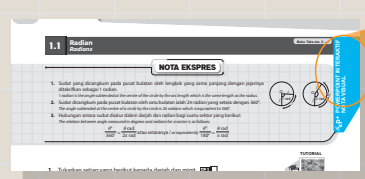
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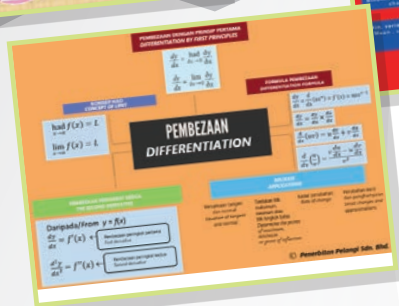
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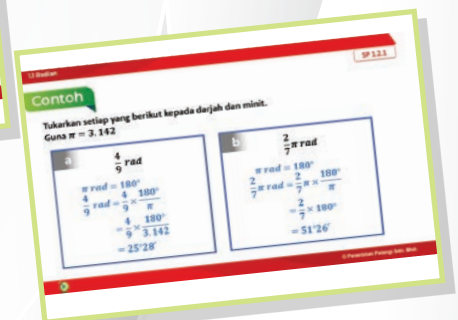
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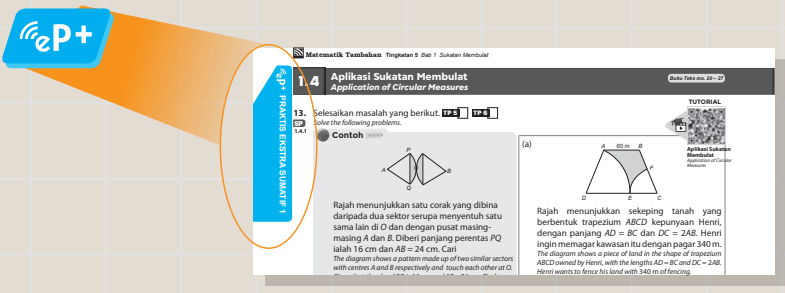
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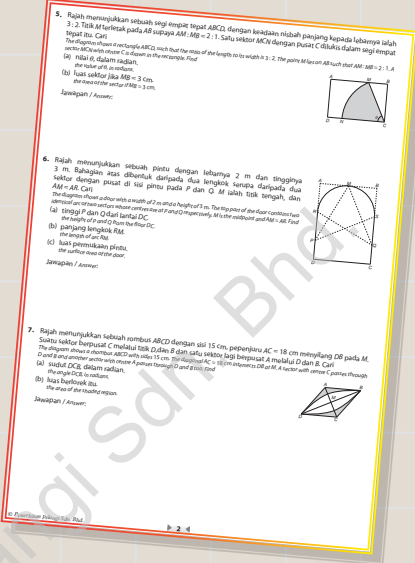
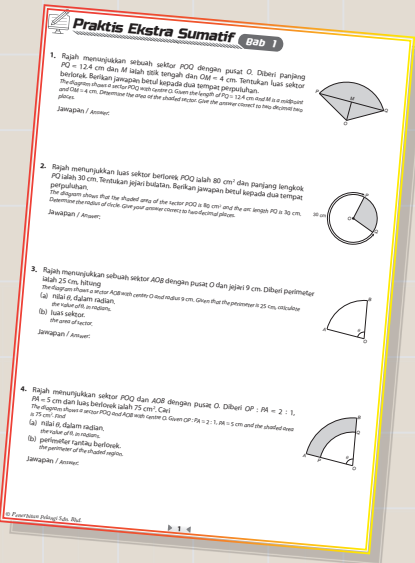
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➤ **PowerPoint Interaktif**
Slaid pengajaran PPT lengkap yang meliputi setiap topik dan subtopik.





Praktis Ekstra Sumatif
Latihan pengukuhan konsep mengikut topik



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▷ Kukuh Kemahiran **Kod QR**

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REKOD PENTAKSIRAN MURID

MATEMATIK TAMBAHAN *Tingkatan 5*

Nama:

Tingkatan:

BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) Menguasai	(x) BELUM Menguasai
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1 Sukatan Membulat	TP1	Mempamerkan pengetahuan asas tentang sukatan membulat.	1 – 2		
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BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) Menguasai	(x) BELUM Menguasai
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pengamiran dalam konteks penyelesaian masalah rutin yang mudah.	35 – 37		
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	TP3	Mengaplikasikan kefahaman tentang pilih atur dan gabungan untuk melaksanakan tugas mudah.	45 – 46		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang pilih atur dan gabungan dalam konteks penyelesaian masalah rutin yang mudah.	46		
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	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah rutin yang mudah.	55 – 56, 59 – 60, 62 – 63, 68 – 70		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah rutin yang kompleks.	56 – 57, 61 – 64		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang taburan kebarangkalian dalam konteks penyelesaian masalah bukan rutin secara kreatif.	58 – 59		
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BAB	STANDARD PRESTASI		HALAMAN	PENCAPAIAN	
	TAHAP PENGUASAAN	TAFSIRAN		(✓) Menguasai	(x) BELUM Menguasai
	TP3	Mengaplikasikan kefahaman tentang fungsi trigonometri untuk melaksanakan tugas mudah.	74 – 75, 82 – 83		
	TP4	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah rutin yang mudah.	75 – 76, 79 – 82, 84 – 85		
	TP5	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah rutin yang kompleks.	76, 85 – 87		
	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang fungsi trigonometri dalam konteks penyelesaian masalah bukan rutin secara kreatif.	77 – 78, 87 – 88		
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8 Kinematik Gerakan Linear	TP1	Mempamerkan pengetahuan asas tentang sesaran, halaju dan pecutan.	104 – 105, 114		
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	TP6	Mengaplikasikan pengetahuan dan kemahiran yang sesuai tentang kinematik gerakan linear dalam konteks penyelesaian masalah bukan rutin secara kreatif.	120 – 123		

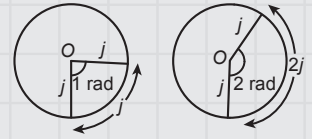
Sukatan Membulat Circular Measure

1.1 Radian Radians

Buku Teks ms. 2 – 4

NOTA EKSPRES

- Sudut yang dicangkum pada pusat bulatan oleh lengkok yang sama panjang dengan jejajarnya ditakrifkan sebagai 1 radian.
1 radian is the angle subtended at the centre of the circle by the arc length which is the same length as the radius.
- Sudut dicangkum pada pusat bulatan oleh satu bulatan ialah 2π radian yang setara dengan 360° .
The angle subtended at the centre of a circle by the circle is 2π radians which is equivalent to 360° .
- Hubungan antara sudut diukur dalam darjah dan radian bagi suatu sektor yang berikut:
The relation between angle measured in degrees and radians for a sector is as follows:



$$\frac{\theta^\circ}{360^\circ} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} \text{ atau setaranya / or equivalently } \frac{\theta^\circ}{180^\circ} = \frac{\theta \text{ rad}}{\pi \text{ rad}}$$

TUTORIAL



Radian
Radians

- Tukarkan setiap yang berikut kepada darjah dan minit. **TP1**

SP Convert each of the following into degrees and minutes.

1.1.1 Guna/Use $\pi = 3.142$.

Contoh

$$\begin{aligned} \frac{4}{3} \pi \text{ rad} &= 1 \pi \text{ rad} = 180^\circ \\ \frac{4}{3} \pi \text{ rad} &= \frac{4}{3} \pi \times \frac{180^\circ}{\pi} \\ &= \frac{4}{3} \times 180^\circ \\ &= 240^\circ \end{aligned}$$

(a) 5.15 rad

$$\begin{aligned} 1 \pi \text{ rad} &= 180^\circ \\ 5.15 \text{ rad} &= 5.15 \times \frac{180^\circ}{\pi} \\ &= 295^\circ 2' \end{aligned}$$

(b) $\frac{3}{7} \pi \text{ rad}$

$$\begin{aligned} 1 \pi \text{ rad} &= 180^\circ \\ \frac{3}{7} \pi \text{ rad} &= \frac{3}{7} \pi \times \frac{180^\circ}{\pi} \\ &= \frac{3}{7} \times 180^\circ \\ &= 77^\circ 9' \end{aligned}$$

- Tukarkan setiap yang berikut kepada radian. **TP1**

SP Convert each of the following into radians.

1.1.1 Guna/Use $\pi = 3.142$.

Contoh

$$\begin{aligned} 319^\circ 14' &= 180^\circ = 1 \pi \text{ rad} \\ 319^\circ 14' &= 319^\circ 14' \times \frac{\pi}{180^\circ} \\ &= 319^\circ 14' \times \frac{3.142}{180^\circ} \\ &= 5.572 \text{ rad} \end{aligned}$$

(a) $134^\circ 24'$

$$\begin{aligned} 180^\circ &= 1 \pi \text{ rad} \\ 134^\circ 24' &= 134^\circ 24' \times \frac{\pi}{180^\circ} \\ &= 134^\circ 24' \times \frac{3.142}{180^\circ} \\ &= 2.346 \text{ rad} \end{aligned}$$

(b) $294^\circ 6'$

$$\begin{aligned} 180^\circ &= 1 \pi \text{ rad} \\ 294^\circ 6' &= 294^\circ 6' \times \frac{\pi}{180^\circ} \\ &= 294^\circ 6' \times \frac{3.142}{180^\circ} \\ &= 5.134 \text{ rad} \end{aligned}$$

1.2 Panjang Lengkok Suatu Bulatan

Arc Length of a Circle

NOTA EKSPRES

- Panjang lengkok, s , suatu bulatan berkadaran dengan sudut yang tercangkum di pusat bulatan.
The arc length, s of a circle is directly proportional to the size of the angle subtended at the centre of the circle.
- Secara am. kita boleh tulis seperti yang berikut:
In general, we can write as follows:

$$\frac{\theta^\circ}{360^\circ} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} = \frac{\text{panjang lengkok (arc length)}}{\text{lilitan bulatan (circumference)}}$$
 dengan panjang lilitan = $2\pi j$ dan jejari ialah j unit.
such that the circumference = $2\pi j$ and the radius is j units.
- Panjang lengkok bulatan, s , dapat ditentukan dengan menggunakan
The arc length of a circle, s , can be determined by using

$$s = j\theta$$
 dengan j ialah jejari bulatan dan θ radian ialah sudut tercangkum oleh lengkok di pusat bulatan.
such that j is the radius of the circle and θ radian is the angle subtended by the arc at the centre of the circle.

3. Tentukan panjang lengkok, s bagi setiap bulatan yang diberi. **TP 1**
- SP** 1.2.1 Guna/Use $\pi = 3.142$.



Contoh

Panjang lengkok, $s = j\theta$
Arc length
 $s = 6.7 \times 4.02$
 $= 26.93 \text{ cm}$

(a)

Panjang lengkok, $s = j\theta$
Arc length
 $s = 0.65 \times 8.2$
 $= 5.33 \text{ cm}$

(b)

$\theta = 2\pi - \frac{5}{3}\pi$
 $= \frac{\pi}{3}$
 Panjang lengkok, $s = j\theta$
Arc length
 $s = \frac{\pi}{3} \times 7.5$
 $= 7.855 \text{ cm}$

(c)

$\theta = 360^\circ - 108^\circ$
 $= 252^\circ$
 $\frac{\theta^\circ}{360^\circ} = \frac{s}{2\pi(4.6)}$
 $s = \frac{252^\circ \times 2\pi(4.6)}{360^\circ}$
 $= 20.23 \text{ cm}$

(d)

$\theta = 2\pi - 0.47\pi$
 $= 1.53\pi$
 Panjang lengkok, $s = j\theta$
Arc length
 $s = 1.53\pi \times 3.22$
 $= 15.48 \text{ cm}$

(e)

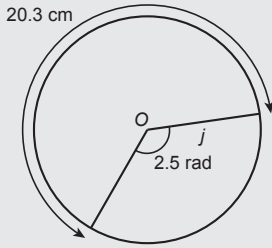
$\theta = (2\pi - 4.76) \text{ rad}$
 $= 1.524 \text{ rad}$
 Panjang lengkok, $s = j\theta$
Arc length
 $s = 1.524 \times 9.8$
 $= 14.94 \text{ cm}$

4. Tentukan jejari bulatan, j , diberikan panjang lengkok dan sudut bagi setiap bulatan yang berikut.

SP 1.2.1

Determine the radius of the circle, j , given the arc length and the angle in each of the circles.

Contoh

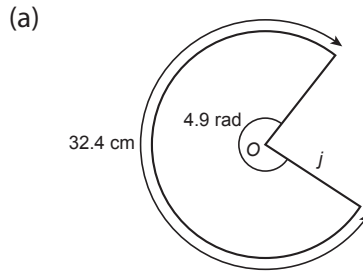


$$s = j\theta$$

$$j = \frac{s}{\theta}$$

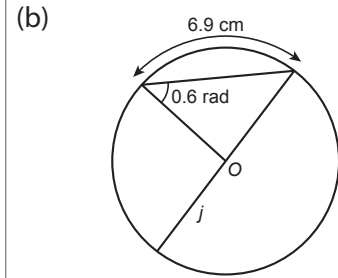
$$j = \frac{20.3}{2\pi - 2.5}$$

$$= 5.36 \text{ cm}$$



$$j = \frac{32.4}{4.9}$$

$$= 6.61 \text{ cm}$$

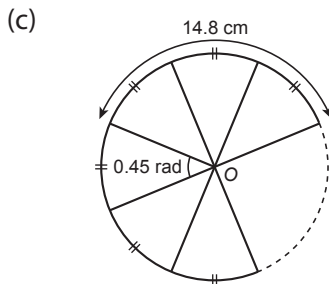


$$s = j\theta$$

$$j \times [\pi - 2(0.6)] = 6.9$$

$$j = \frac{6.9}{\pi - 2(0.6)}$$

$$= 3.55 \text{ cm}$$

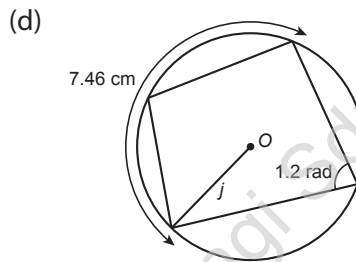


$$s = j\theta$$

$$j \times [3(0.45)] = 14.8$$

$$j = \frac{14.8}{3(0.45)}$$

$$= 10.96 \text{ cm}$$

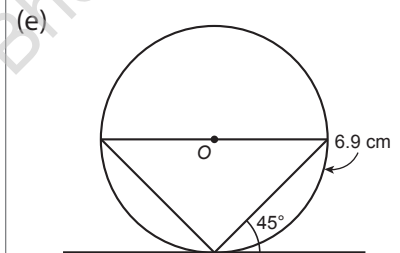


$$s = j\theta$$

$$j \times [2(1.2)] = 7.46$$

$$j = \frac{7.46}{2.4}$$

$$= 3.11 \text{ cm}$$



$$45^\circ = \frac{\pi}{4}$$

$$j \left[2 \left(\frac{\pi}{4} \right) \right] = 6.9$$

$$j = \frac{6.9}{\frac{\pi}{2}}$$

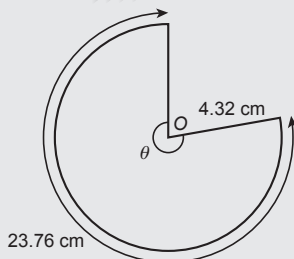
$$= 4.39 \text{ cm}$$

5. Tentukan sudut tercangkum, θ dalam radian, di pusat bulatan dengan diberikan jejari bulatan dan panjang lengkok bagi setiap bulatan yang berikut.

SP 1.2.1

Determine the subtended angle, θ , in radians, at the centre of the circle given that the radius and the arc length of each of the following circles.

Contoh

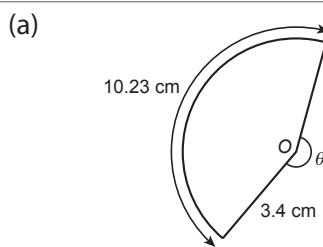


$$s = j\theta$$

$$\theta = \frac{s}{j}$$

$$\theta = \frac{23.76}{4.32}$$

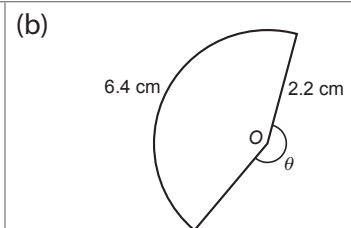
$$= 5.5 \text{ rad}$$



$$2\pi - \theta = \frac{10.23}{3.4}$$

$$\theta = 2\pi - 3$$

$$= 3.284 \text{ rad}$$



$$2\pi - \theta = \frac{6.4}{2.2}$$

$$\theta = 2\pi - 2.9$$

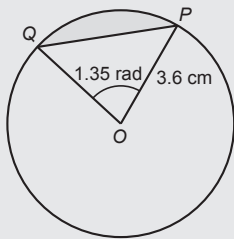
$$= 3.384 \text{ rad}$$

6. Tentukan perimeter tembereng yang berlorek bagi setiap bulatan berpusat O yang berikut.

TP 4

SP 1.2.2 Determine the perimeter of the shaded segment of each of the following circles with centre O.

Contoh



Tip Penting

Perentas PQ dapat diperolehi dengan petua kosinus, iaitu $PQ = \sqrt{j^2 + j^2 - 2j^2 \cos \theta}$, dengan θ dalam darjah.
The chord PQ can be obtained by using the cosine rule, that is $PQ = \sqrt{j^2 + j^2 - 2j^2 \cos \theta}$, such that θ is in degrees.

$$1.35 \text{ rad} = 1.35 \times \frac{180^\circ}{\pi} = 77.34^\circ$$

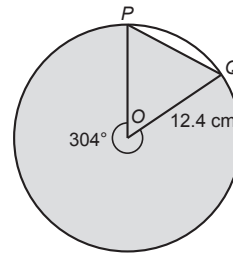
Maka/Hence,

$$PQ = \sqrt{3.6^2 + 3.6^2 - 2(3.6)^2 \cos / \cos 77.34^\circ} = 4.5 \text{ cm}$$

Panjang lengkok PQ = $j\theta$
 Arc length of PQ
 $= 3.6 \times 1.35 = 4.86 \text{ cm}$

Perimeter tembereng berlorek
 Perimeter of the shaded segment
 $= (4.5 + 4.86) \text{ cm} = 9.36 \text{ cm}$

(a)



$$56^\circ = 56^\circ \times \frac{\pi}{180^\circ} = 0.978 \text{ rad}$$

Panjang lengkok minor PQ = $j\theta$
 Minor arc length of PQ
 $= 12.4 \times 0.978 = 12.13 \text{ cm}$

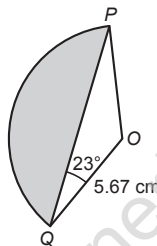
Panjang lengkok major PQ
 Major arc length of PQ
 $= 2\pi(12.4) - 12.13 = 65.79 \text{ cm}$

Maka/Hence,

$$PQ = \sqrt{12.4^2 + 12.4^2 - 2(12.4)^2 \cos / \cos 56^\circ} = 11.64 \text{ cm}$$

Perimeter tembereng berlorek
 Perimeter of the shaded segment
 $= 11.64 + 65.79 = 77.43 \text{ cm}$

(b)



$$\angle POQ = 180^\circ - 2(23^\circ) = 134^\circ$$

Maka/Hence,

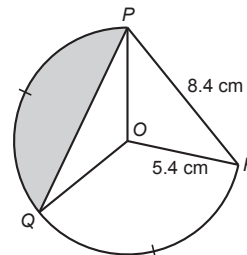
$$PQ = \sqrt{5.67^2 + 5.67^2 - 2(5.67)^2 \cos / \cos 134^\circ} = 10.44 \text{ cm}$$

$$134^\circ = 134 \times \frac{\pi}{180^\circ} = 2.34 \text{ rad}$$

Panjang lengkok PQ
 Arc length of PQ
 $= 2.34 \times 5.67 = 13.27 \text{ cm}$

Perimeter tembereng berlorek
 Perimeter of the shaded segment
 $= 13.27 + 10.44 = 23.71 \text{ cm}$

(c)



$$PR = \sqrt{5.4^2 + 5.4^2 - 2(5.4)^2 \cos / \cos \theta}$$

$$8.4^2 = 5.4^2 + 5.4^2 - 2(5.4)^2 \cos / \cos \theta$$

$$\theta = \cos^{-1} \left[\frac{5.4^2 + 5.4^2 - 8.4^2}{2(5.4)^2} \right] = 102.12^\circ = 1.783 \text{ rad}$$

$$PQ = \sqrt{5.4^2 + 5.4^2 - 2(5.4)^2 \cos / \cos 128.94^\circ} = 9.745 \text{ cm}$$

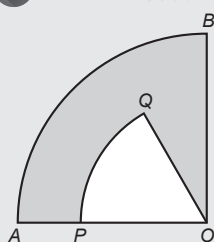
Panjang lengkok PQ
 Arc length of PQ
 $= 5.4 \times 2.25 = 12.15 \text{ cm}$

Perimeter tembereng berlorek
 Perimeter of the shaded segment
 $= 9.745 \text{ cm} + 12.15 \text{ cm} = 21.895 \text{ cm}$

7. Selesaikan masalah yang melibatkan panjang lengkok.

SP 1.2.3 Solve the problems involving the arc lengths.

Contoh



Rajah menunjukkan dua sektor, AOB dan POQ berpusat O. Diberi bahawa $OQ : OB = 2 : 3$, $OP = j$ cm, $\angle AOB : \angle POQ = 3 : 2$ dan $\angle AOB = \theta$ rad. Cari perimeter rantau berlorek dalam sebutan θ dan j .

The diagram shows two sectors, AOB and POQ with centre O. Given that $OQ : OB = 2 : 3$, $OP = j$ cm, $\angle AOB : \angle POQ = 3 : 2$ and $\angle AOB = \theta$ rad. Find the perimeter of the shaded region in terms of θ and j .

Panjang $OB = \frac{3}{2}j$ cm
Length of OB

Panjang lengkok $PQ = j\left(\frac{2}{3}\right)\theta = \frac{2}{3}j\theta$ cm
Arc length of PQ

Panjang lengkok $AB = j\left(\frac{3}{2}\right)\theta = \frac{3}{2}j\theta$ cm
Arc length of AB

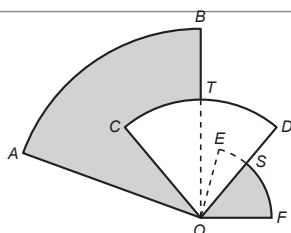
Perimeter rantau berlorek

Perimeter of the shaded region
 $= \frac{1}{2}j + \frac{2}{3}j\theta + j + \frac{3}{2}j + \frac{3}{2}j\theta$

$= 2j + \frac{13}{6}j\theta$

$= \left(2 + \frac{13}{6}\theta\right)j$ cm

(a)



Rajah menunjukkan tiga sektor, AOB, COD dan EOF berpusat O. Nisbah jejari $OA : OC : OE = 8 : 5 : 3$. Titik T membahagikan lengkok CD kepada dua bahagian yang sama manakala titik S pada lengkok EF membahagikan lengkok EF dengan nisbah $1 : 2$. Jika sudut terancang oleh ketiga-tiga lengkok adalah sama dan bernilai 1.5 rad dan jejari $OE = 6$ cm. Cari perimeter kawasan yang berlorek itu.

The diagram shows three sectors, AOB, COD and EOF with centre O. The ratio of the radii is $OA : OC : OE = 8 : 5 : 3$. The point T divides the arc CD into two equal parts while point S divides the arc EF in the ratio of $1 : 2$. If the angle subtended by the three arcs are the same and has a value of 1.5 rad and the radius $OE = 6$ cm. Find the perimeter of the shaded region.

$OA : OC : OE = 8 : 5 : 3$

$ES : SF = 1 : 2$

Perimeter kawasan berlorek

Perimeter of the shaded region

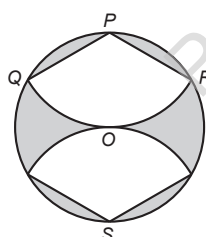
$= OF + \text{panjang lengkok } SF + OS + OC + \text{panjang lengkok } CT + TB + \text{panjang lengkok } AB + OA$

$= 6 + (1)(6) + 6 + 10 + 0.75(10) + (16 - 10) + 1.5(16) + 16$

$= 6 + 6 + 6 + 10 + 7.5 + 6 + 24 + 16$

$= 81.5$ cm

(b)



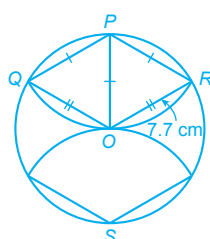
Rajah menunjukkan sebuah bulatan berpusat O bersama dengan sebuah sektor berpusat P yang terletak pada lilitan bulatan. Dua sektor serupa dengan pusat masing-masing pada P dan S menyentuh antara satu sama lain pada O. Diberi bahawa jejari bulatan ialah 7.7 cm, cari

The diagram shows a circle with centre O and a sector with centre P which is on the circumference of the circle. Two similar sectors with centres P and S touch each other at O respectively. Given that the radius of the circle is 7.7 cm, find

(i) sudut QPR , dalam radian.
the angle QPR , in radians.

(ii) perimeter rantau berlorek, dalam cm.
the perimeter of the shaded region, in cm.

(i)



$\angle QPR = 120^\circ$

$120^\circ = 120 \times \frac{\pi}{180} = 2.095$ rad

(ii) Perimeter rantau berlorek

Perimeter of the shaded region

$= 2\pi(7.7) + 2\left[2(7.7) + 7.7\left(\frac{2\pi}{3}\right)\right]$

$= 15.4\pi + 2(15.4 + 5.133\pi)$

$= 15.4\pi + 30.8 + 10.266\pi$

$= 25.666\pi + 30.8$

$= 111.44$ cm

1.3

Luas Sektor Suatu Bulatan Area of Sector of a Circle

NOTA EKSPRES

1. Hubungan secara am antara sudut, panjang lengkok dan luas sektor adalah seperti berikut,
The relationship between angles, arc length and the area are as follows

$$\frac{\theta^\circ}{360^\circ} = \frac{\theta \text{ rad}}{2\pi \text{ rad}} = \frac{\text{panjang lengkok / arc length, } s}{2\pi j} = \frac{\text{luas sektor / area of sector}}{\text{luas bulatan / area of circle}}$$

2. Luas sektor bulatan, L , dapat ditentukan dengan menggunakan
The area of a sector of the circle, L , can be determined by using

$$L = \frac{1}{2}j^2\theta$$

dengan j ialah jejari bulatan dan θ radian ialah sudut tercangkum oleh sektor di pusat bulatan.

where j is the radius of the circle and θ radian is the angle subtended by the sector at the centre of the circle.

TUTORIAL



Luas Sektor Suatu Bulatan
Sector Area of a Circle

INFO



Cara mencari rumus panjang perentas, luas sektor dan luas tembereng.

Steps to find the length of chord, the area of sector and the area of segment.

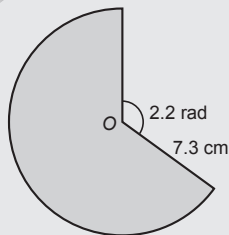
8. Tentukan luas sektor berlorek bagi setiap bulatan yang berikut. Beri jawapan anda betul kepada dua tempat perpuuhan.

SP
1.3.1

TP 3

Determine the area of the shaded sector for each of the following circles. Give your answer correct to two decimal places.

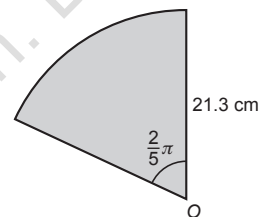
Contoh



Sudut tercangkum = $(2\pi - 2.2)$ rad
Subtended angle

$$\begin{aligned} L &= \frac{1}{2}j^2\theta \\ &= \frac{1}{2}(7.3)^2(2\pi - 2.2) \\ &= \frac{1}{2} \times 53.29 \times 4.084 \\ &= 108.82 \text{ cm}^2 \end{aligned}$$

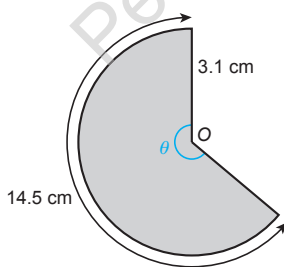
(a)



Sudut tercangkum = $\frac{2}{5}\pi$ rad
Subtended angle

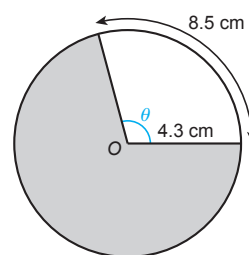
$$\begin{aligned} L &= \frac{1}{2}j^2\theta \\ &= \frac{1}{2}(21.3)^2\left(\frac{2\pi}{5}\right) \\ &= \frac{1}{2} \times 453.69 \times 1.2568 \\ &= 285.09 \text{ cm}^2 \end{aligned}$$

(b)



$$\begin{aligned} s &= j\theta \\ 14.5 &= 3.1\theta \\ \theta &= 4.677 \text{ rad} \\ L &= \frac{1}{2}j^2\theta \\ &= \frac{1}{2}(3.1)^2(4.677) \\ &= \frac{1}{2} \times 9.61 \times 4.677 \\ &= 22.47 \text{ cm}^2 \end{aligned}$$

(c)



$$\begin{aligned} s &= j\theta \\ 8.5 &= 4.3\theta \\ \theta &= 1.977 \text{ rad} \\ \text{Sudut tercangkum} &= (2\pi - 1.977) \text{ rad} \\ \text{Subtended angle} \\ L &= \frac{1}{2}j^2\theta \\ &= \frac{1}{2}(4.3)^2(2\pi - 1.977) \\ &= \frac{1}{2} \times 18.49 \times 4.307 \\ &= 39.82 \text{ cm}^2 \end{aligned}$$

9. Tentukan jejari bagi sektor bulatan berlorek yang berikut. Beri jawapan betul kepada dua tempat perpuluhan.

SP
1.3.1

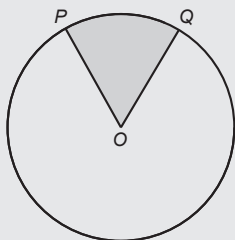
Determine the radius of the following shaded sectors of circles. Give your answer correct to two decimal places.

TP 4

Contoh

Diberi bahawa luas sektor ialah 43 cm^2 dan panjang lengkok PQ ialah 9.5 cm .

Given that the area of the sector is 43 cm^2 and the arc length of PQ is 9.5 cm .



$$\frac{9.5}{2\pi j} = \frac{43}{\pi j^2}$$

$$9.5 \times \pi j^2 = 43 \times 2\pi j$$

$$9.5\pi j^2 = 86\pi j$$

$$j = \frac{86}{9.5}$$

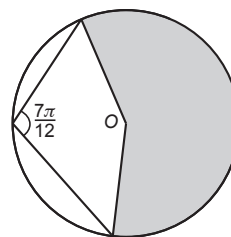
$$= 9.05 \text{ cm}$$

Tip Penting

Guna $\frac{\text{luas sektor}}{\text{luas bulatan}} = \frac{\theta}{2\pi}$

Use $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\theta}{2\pi}$

(a) Diberi bahawa luas sektor berpusat O ialah 56 cm^2 .
Given that the area of the sector with centre O is 56 cm^2 .



$$\theta = \frac{7\pi}{12} \times 2 = \frac{7\pi}{6}$$

$$\frac{1}{2}j^2 \left(\frac{7\pi}{6} \right) = 56$$

$$j^2 = \frac{56 \times 12}{7\pi} = \frac{96}{\pi}$$

$$j = 5.53 \text{ cm}$$

(b) Diberi bahawa luas sektor A ialah empat kali luas sektor B apabila sudut yang tercangkum adalah sama. Tunjukkan bahawa jejari bagi sektor A adalah dua kali panjang jejari bagi sektor B .

Given that the area of sector A is four times the area of sector B when the subtended angle is the same. Show that the radius of sector A is twice the length of the radius of sector B .

$$\frac{1}{2}(j_1)^2\theta : \frac{1}{2}(j_2)^2\theta = 4 : 1$$

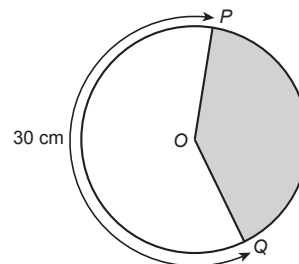
$$\frac{j_1^2}{j_2^2} = \frac{4}{1}$$

$$\frac{j_1}{j_2} = \frac{2}{1}$$

$$j_1 = 2j_2$$

(Tertunjuk/Shown)

(c)



Diberi bahawa luas sektor berlorek ialah 80 cm^2 dan panjang lengkok major PQ ialah 30 cm .

Given that the area of the shaded sector is 80 cm^2 and the major arc length PQ is 30 cm .

Tip Penting

Guna/ Use

$\frac{\text{panjang lengkok lilitan bulatan}}{\text{luas bulatan}} = \frac{\text{luas sektor}}{\text{luas bulatan}}$

$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{area of sector}}{\text{area of circle}}$

$$\frac{30}{2\pi j} = \frac{\pi j^2 - 80}{\pi j^2}$$

$$j = \frac{2(\pi j^2 - 80)}{30}$$

$$\pi j^2 - 80 = 15j$$

$$\pi j^2 - 15j - 80 = 0$$

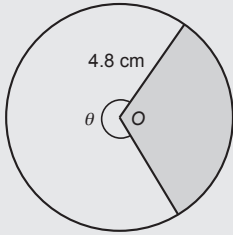
$$j = \frac{15 \pm \sqrt{(-15)^2 - 4\pi(-80)}}{2\pi}$$

$$j = 7.97 \text{ cm}$$

10. Tentukan sudut terancang, θ dalam radian, di pusat bulatan bagi setiap yang berikut. Beri jawapan betul kepada dua tempat perpuluhan jika perlu. TP 4

1.3.1 Determine the subtended angle, θ in radians, at the centre of the circle for each of the following. Give your answer correct to two decimal places where necessary.

Contoh



Diberi bahawa luas sektor berlorek ialah 23 cm^2 .
Given that the area of the shaded sector is 23 cm^2 .

$$\frac{23}{\pi(4.8)^2} = \frac{2\pi - \theta}{2\pi}$$

$$23 \times 2\pi = (2\pi - \theta) \times \pi(4.8)^2$$

$$\theta = 2\pi - \frac{23 \times 2\pi}{(4.8)^2}$$

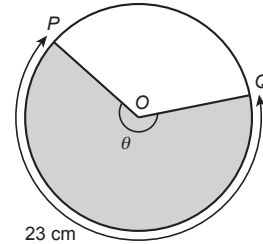
$$= 4.29 \text{ radian/radians}$$

Tip Penting

Guna $\frac{\text{luas sektor}}{\text{luas bulatan}} = \frac{2\pi - \theta}{2\pi}$

Use $\frac{\text{area of sector}}{\text{area of circle}} = \frac{2\pi - \theta}{2\pi}$

(a) Diberi bahawa luas sektor berlorek ialah 65 cm^2 dan panjang lengkok major PQ ialah 23 cm .
Given that the area of the shaded sector is 65 cm^2 and the major arc length PQ is 23 cm .



$$\frac{65}{\pi j^2} = \frac{23}{2\pi j}$$

$$j = \frac{65 \times 2}{23}$$

$$= 5.65 \text{ cm}$$

$$s = j\theta$$

$$\theta = \frac{23}{5.65}$$

$$= 4.07 \text{ radian/radians}$$

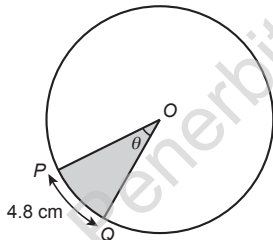
Tip Penting

Guna/ Use

$\frac{\text{panjang lengkok lilitan bulatan}}{\text{arc length}} = \frac{\text{luas sektor}}{\text{luas bulatan}}$

$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{area of sector}}{\text{area of circle}}$

(b) Diberi bahawa luas sektor berlorek ialah 19.2 cm^2 dan panjang lengkok PQ ialah 4.8 cm .
Given that the area of the shaded sector is 19.2 cm^2 and the arc length PQ is 4.8 cm .



$$\frac{1}{2}j^2\theta = 19.2$$

$$j^2\theta = 38.4$$

$$j\theta = 4.8$$

$$j(j\theta) = 38.4$$

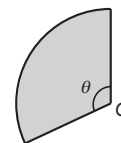
$$j(4.8) = 38.4$$

$$j = 8$$

$$\theta = \frac{s}{j}$$

$$\theta = \frac{4.8}{8} = 0.6 \text{ radian}$$

(c) Diberi bahawa luas suatu sektor ialah 49 cm^2 dan perimetranya ialah 28 cm .
Given that the area of the sector is 49 cm^2 and the perimeter is 28 cm .



$$\frac{1}{2}j^2\theta = 49 \dots\dots \textcircled{1}$$

$$j + j + j\theta = 28 \dots\dots \textcircled{2}$$

$$\theta = \frac{49}{\frac{1}{2}j^2} = \frac{98}{j^2}$$

Daripada/From $\textcircled{2}$,

$$2j + j\left(\frac{98}{j^2}\right) = 28$$

$$2j^2 - 28j + 98 = 0$$

$$(j - 7)(j - 7) = 0$$

$$j = 7 \text{ cm}$$

$$\theta = \frac{98}{7^2} = 2 \text{ rad}$$

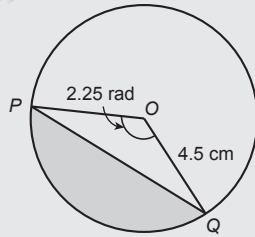
11. Tentukan luas tembereng yang berlorek bagi setiap bulatan yang berikut. Beri jawapan betul kepada dua tempat perpuluhan.

SP
1.3.2

TP 5

Determine the area of the shaded segment for each of the following circles. Give your answer correct to two decimal places.

Contoh



Tip Penting

Luas tembereng berlorek/ Area of shaded segment
= luas sektor POQ – luas segi tiga POQ
area of sector POQ – area of triangle POQ

Luas sektor POQ / Area of sector POQ

$$= \frac{1}{2} (4.5)^2 (2.25)$$

$$= 22.78 \text{ cm}^2$$

$$2.25 \text{ rad} = \frac{2.25 \times 180^\circ}{\pi} = 128.9^\circ$$

Luas segi tiga POQ / Area of triangle POQ

$$= \frac{1}{2} (4.5)^2 \sin 128.9^\circ$$

$$= 7.88 \text{ cm}^2$$

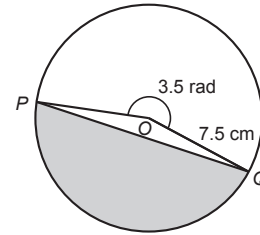
Maka, luas tembereng berlorek

Hence, the area of shaded segment

$$= 22.78 - 7.88$$

$$= 14.9 \text{ cm}^2$$

(a)



$$\angle_{\text{minor}} POQ = (2\pi - 3.5) \text{ rad}$$

Luas sektor minor POQ / Area of minor sector POQ

$$= \frac{1}{2} (7.5)^2 (2\pi - 3.5)$$

$$= \frac{1}{2} \times 56.25 \times 2.784$$

$$= 78.3 \text{ cm}^2$$

$$(2\pi - 3.5) \text{ rad} = \frac{2.784 \times 180^\circ}{\pi} = 159.5^\circ$$

Luas segi tiga POQ / Area of triangle POQ

$$= \frac{1}{2} (7.5)^2 \sin 159.5^\circ$$

$$= 9.85 \text{ cm}^2$$

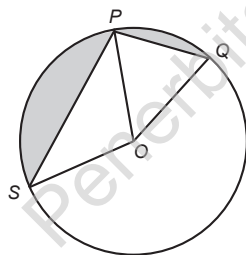
Maka, luas tembereng berlorek

Hence, the area of the shaded segment

$$= 78.3 - 9.85$$

$$= 68.45 \text{ cm}^2$$

(b)



Diberi bahawa POQ dan POS adalah dua sektor dengan pusat O, dan jejari bulatan ialah 4.5 cm, dengan keadaan $\angle POQ = \frac{1}{2} \angle POS = 0.9 \text{ rad}$.

Given that POQ and POS are two sectors with centre O, and the radius of the circle is 4.5 cm, such that

$$\angle POQ = \frac{1}{2} \angle POS = 0.9 \text{ rad.}$$

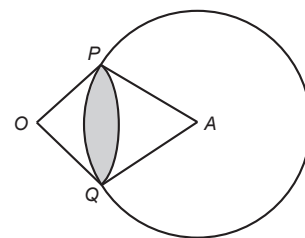
Luas sektor POS / Area of sector POS

$$= \frac{1}{2} r^2 \theta = \frac{1}{2} (4.5)^2 (1.8)$$

$$= 18.225 \text{ cm}^2$$

$$1.8 \text{ rad} = \frac{1.8 \times 180^\circ}{3.142} = 103.12^\circ$$

(c)



Diberi bahawa sebuah sektor POQ berpusat O, dan jejari OP = 6 cm. P dan Q adalah pada lilitan bulatan berpusat A, dengan keadaan $\angle PAQ = 65^\circ$ dan PA = 7.5 cm.

Given that the sector POQ with centre O, and radius OP = 6 cm. P and Q are on the circumference of the circle with centre A, such that $\angle PAQ = 65^\circ$ and PA = 7.5 cm.

Perentas PQ / The chord PQ

$$= \sqrt{7.5^2 + 7.5^2 - 2(7.5)^2 \cos / \cos 65^\circ}$$

$$= 8.06 \text{ cm}$$

Luas segi tiga POS / Area of triangle POS

$$= \frac{1}{2} (4.5)^2 \sin 103.12^\circ$$

$$= 9.86 \text{ cm}^2$$

Maka, luas tembereng berlorek PS

Hence, the area of the shaded segment PS

$$= 18.225 - 9.86$$

$$= 8.365 \text{ cm}^2$$

Luas sektor POQ / Area of sector POQ

$$= \frac{1}{2} r^2 \theta = \frac{1}{2} (4.5)^2 (0.9)$$

$$= 9.11 \text{ cm}^2$$

$$0.9 \text{ rad} = \frac{0.9 \times 180^\circ}{3.142} = 51.56^\circ$$

Luas segi tiga POQ / Area of triangle POQ

$$= \frac{1}{2} (4.5)^2 \sin 51.56^\circ$$

$$= 7.93 \text{ cm}^2$$

Maka, luas tembereng berlorek PQ

Hence, the area of the shaded segment PQ

$$= 9.11 - 7.93$$

$$= 1.18 \text{ cm}^2$$

Jumlah luas tembereng berlorek

Total area of the shaded segment

$$= 8.365 + 1.18$$

$$= 9.545 \text{ cm}^2$$

$$\cos/\cos \angle POQ = \frac{6^2 + 6^2 - 8.06^2}{2(6)^2}$$

$$= 0.09773$$

$$\angle POQ = 84.39^\circ$$

$$84.39^\circ = \frac{84.39^\circ \times 3.142}{180^\circ}$$

$$= 1.4731 \text{ rad}$$

$$65^\circ = \frac{65^\circ \times 3.142}{180^\circ}$$

$$= 1.1346 \text{ rad}$$

Luas tembereng berlorek

The area of the shaded segment

$$= \left[\frac{1}{2} \times (6)^2 \times (1.4731) \right] - \left[\frac{1}{2} \times (6)^2 \times \sin 84.39^\circ \right]$$

$$+ \left[\frac{1}{2} \times (7.5)^2 \times (1.1346) \right] - \left[\frac{1}{2} \times (7.5)^2 \times \sin 65^\circ \right]$$

$$= 15.02 \text{ cm}^2$$

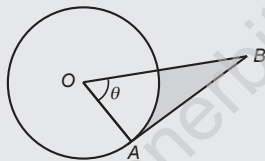
12. Selesaikan yang berikut.

SP Solve the following.

1.3.3

TP 5

Contoh

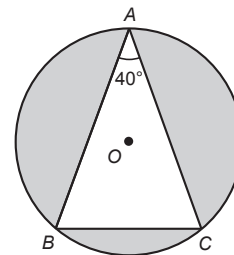


Rajah menunjukkan sebuah bulatan dengan pusat O dan jejari 5 cm. Diberi bahawa AB ialah tangen kepada bulatan pada A dan $OB = 10$ cm. Cari

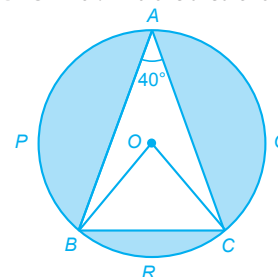
The diagram shows a circle with centre O and radius 5 cm. Given that AB is a tangent to the circle at A and $OB = 10$ cm. Find

- (i) nilai θ , dalam radian.
the value of θ , in radians.
- (ii) luas rantau berlorek.
the area of the shaded region.

(a)



Rajah menunjukkan sebuah bulatan dengan pusat O dan jejari 5 cm. A, B dan C terletak pada lilitan bulatan, dengan keadaan $AB = AC$. Diberi bahawa $\angle BAC = 40^\circ$. Cari luas rantau berlorek itu.
The diagram shows a circle with centre O and radius 5 cm. A, B and C lie on the circumference of the circle, such that $AB = AC$. Given that $\angle BAC = 40^\circ$. Find the area of the shaded region.

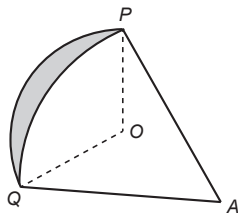


(i) $\cos/\cos \theta = \frac{5}{10}$
 $\theta = \frac{\pi}{3}$ rad

(ii) Luas rantau berlorek/Area of the shaded region
 $= \frac{1}{2}(5)(10) \sin 60^\circ - \frac{1}{2}(5)^2 \left(\frac{\pi}{3}\right)$
 $= 21.6506 - 13.0917$
 $= 8.56 \text{ cm}^2$

$\angle BOC = 80^\circ$
 $\angle AOB = 180^\circ - 40^\circ = 140^\circ$
 Luas rantau berlorek APB
 $=$ luas rantau berlorek AQC
 Area of the shaded region APB
 $=$ Area of the shaded region AQC
 Jumlah luas rantau berlorek
 Total area of the shaded region
 $= 2 \left[\frac{140^\circ}{360^\circ} \times \pi(5)^2 - \frac{1}{2}(5)^2 \sin 140^\circ \right] +$
 $\left[\frac{80^\circ}{360^\circ} \times \pi(5)^2 - \frac{1}{2}(5)^2 \sin 80^\circ \right]$
 $= 45.025 + 5.145$
 $= 50.17 \text{ cm}^2$

(b)



Rajah menunjukkan dua buah sektor dengan pusat O dan A masing-masing menyilang pada titik P dan Q. Diberi bahawa $OP = 8 \text{ cm}$, $\angle POQ = \frac{2\pi}{3}$ rad dan $AP = 15 \text{ cm}$. Cari

The diagram shows two sectors with centres O and A, that intersect at points P and Q respectively. Given that $OP = 8 \text{ cm}$, $\angle POQ = \frac{2\pi}{3}$ rad and $AP = 15 \text{ cm}$. Find

(i) $\angle PAQ$ dalam radian.
 $\angle PAQ$ in radians.

(ii) luas rantau berlorek.
 the area of the shaded region.

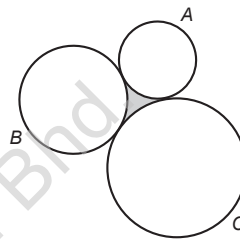
(i) $\angle POQ = \frac{2\pi}{3}$ rad $= 120^\circ$
 Perentas PQ / Chord PQ
 $= \sqrt{8^2 + 8^2 - 2(8)^2 \cos/\cos 120^\circ}$
 $= 13.856 \text{ cm}$

$\cos/\cos \angle PAQ = \frac{15^2 + 15^2 - 13.856^2}{2(15)^2}$
 $= 0.57336$

$55.02^\circ = \frac{55.02^\circ \times 3.142}{180^\circ} = 0.9604 \text{ rad}$

(ii) Luas rantau berlorek/Area of the shaded region
 $= \left[\frac{1}{2}(8)^2 \left(\frac{2\pi}{3}\right) - \frac{1}{2}(8)^2 \sin 120^\circ \right] -$
 $\left[\frac{1}{2}(15)^2(0.9604) - \frac{1}{2}(15)^2 \sin 55.02^\circ \right]$
 $= 23.45 \text{ cm}^2$

(c)



Rajah menunjukkan tiga bulatan, A, B dan C yang menyentuh antara satu sama lain. Jejari bagi bulatan dari yang terkecil ialah 5 cm, 7 cm dan 9 cm. Cari luas rantau yang berlorek.

The diagram shows three circles, A, B and C that touch each other. The radius of the circles from the smallest are 5 cm, 7 cm and 9 cm. Find the area of the shaded region.

$PQ = 5 + 7 = 12 \text{ cm}$
 $PR = 7 + 9 = 16 \text{ cm}$
 $QR = 5 + 9 = 14 \text{ cm}$

$\cos/\cos \angle QPR = \frac{12^2 + 16^2 - 14^2}{2(12)(16)}$
 $\angle QPR = 57.91^\circ$

$57.91^\circ = \frac{57.91^\circ \times 3.142}{180^\circ} = 1.011 \text{ rad}$
 $\frac{\sin \angle R}{12} = \frac{\sin 57.91^\circ}{14}$

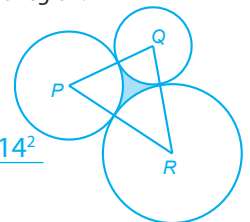
$\sin \angle R = 0.7262$
 $\angle R = 46.57^\circ$

$46.57^\circ = \frac{46.57^\circ \times 3.142}{180^\circ} = 0.813 \text{ rad}$
 $\angle Q = 180^\circ - 46.57^\circ - 57.91^\circ = 75.52^\circ$

$75.52^\circ = \frac{75.52^\circ \times 3.142}{180^\circ} = 1.318 \text{ rad}$

Luas rantau berlorek = luas segi tiga PQR - tiga sektor dalam segi tiga itu
 Area of the shaded region = area of triangle PQR - three sectors in the triangle.

$= \frac{1}{2}(12)(16) \sin 57.91^\circ - \frac{1}{2}(7)^2(1.011)$
 $- \frac{1}{2}(5)^2(1.318) - \frac{1}{2}(9)^2(0.813)$
 $= 7.159 \text{ cm}^2$

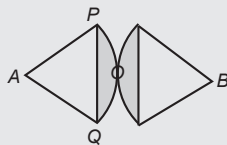


1.4 Aplikasi Sukatan Membulat Application of Circular Measures

13. Selesaikan masalah yang berikut. **TP 5** **TP 6**
SP Solve the following problems.

1.4.1

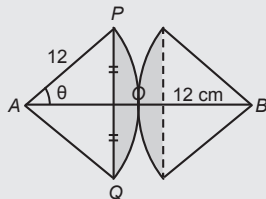
Contoh



Rajah menunjukkan satu corak yang dibina daripada dua sektor serupa menyentuh satu sama lain di O dan dengan pusat masing-masing A dan B . Diberi panjang perentas PQ ialah 16 cm dan $AB = 24$ cm. Cari
 The diagram shows a pattern made up of two similar sectors with centres A and B respectively and touch each other at O . Given that the chord PQ is 16 cm and $AB = 24$ cm. Find

- (i) $\angle PAQ$ dalam radian.
 $\angle PAQ$ in radians.
- (ii) perimeter rantau berlorek.
the perimeter of the shaded region.
- (iii) luas rantau berlorek.

PENYELESAIAN



- (i) $\sin \theta = \frac{8}{12}$
 $= 41.81^\circ = 0.73 \text{ rad}$
 $\angle PAQ = 1.46 \text{ rad}$
- (ii) Perimeter rantau berlorek
Perimeter of the shaded region
 $= 2[16 + 12(1.46)]$
 $= 67.04 \text{ cm}$
- (iii) Luas rantau berlorek
Area of shaded region
 $= 2\left[\frac{1}{2}(12)^2(1.46) - \frac{1}{2}(12)^2 \sin 83.62^\circ\right]$
 $= 67.13 \text{ cm}^2$

PROJEK STEM



Aplikasi sukatan membulat
Application of circular measures

AKTIVITI PAK-21



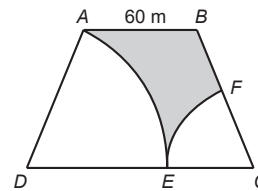
Perimeter dan luas tembereng
Perimeter and area of the segment

TUTORIAL



Aplikasi Sukatan Membulat
Application of Circular Measures

(a)



Rajah menunjukkan sekeping tanah yang berbentuk trapezium $ABCD$ kepunyaan Henri, dengan panjang $AD = BC$ dan $DC = 2AB$. Henri ingin memagar kawasan itu dengan pagar 340 m. The diagram shows a piece of land in the shape of trapezium $ABCD$ owned by Henri, with the lengths $AD = BC$ and $DC = 2AB$. Henri wants to fence his land with 340 m of fencing.

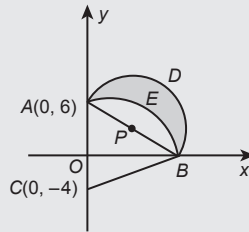
- (i) Cari $\angle ADC$, dalam radian. **KBAT** *Menganalisis*
Find $\angle ADC$, in radians.
- (ii) Sektor ADE berpusat D dan sektor ECF berpusat C akan digunakan untuk menanam sayur dan bahagian yang tertinggal diliputi dengan rumput. Cari luas yang diliputi dengan rumput
The sectors ADE with centre D and ECF with centre C will be used to plant vegetables, and the remaining will be covered with grass. Find the area covered by grass.

- (i) Panjang AD / Length of AD
 $= \frac{340 - 60 - 120}{2} = 80 \text{ m}$
 $\cos / \cos \angle ADC = \frac{30}{80}$
 $\angle ADC = 67.98^\circ$
 $67.98^\circ = \frac{67.98 \times 3.142}{180} = 1.1866 \text{ rad}$
 $\angle ADC = 1.1866 \text{ rad}$
- (ii) Luas trapezium $ABCD$ / Area of trapezium $ABCD$
 $= \frac{1}{2}(60 + 120)(80 \sin 67.98^\circ)$
 $= 6674.78 \text{ m}^2$
 Luas sektor ADE / Area of sector ADE
 $= \frac{1}{2}(80)^2(1.1866)$
 $= 3797.12 \text{ m}^2$
 Luas sektor ECF / Area of sector ECF
 $= \frac{1}{2}(40)^2(1.1866)$
 $= 949.28 \text{ m}^2$
 Luas diliputi rumput / Area covered by grass
 $= 6674.78 - 3797.12 - 949.28$
 $= 1928.38 \text{ m}^2$

KBAT EKSTRA



MASTERI KE ARAH SPM



Rajah menunjukkan satu semibulatan $PADB$ berpusat P dan sektor CAB berpusat C dilukis pada satah Cartes. Diberi $CP = 5\sqrt{3}$ unit, cari

The diagram shows a semicircle $PADB$ with centre P and a sector CAB with centre C drawn on a Cartesian plane. Given that $CP = 5\sqrt{3}$ units, find

- (i) diameter semibulatan itu.
the diameter of the semicircle.
- (ii) sudut ACB , dalam radian.
the angle ACB , in radians.
- (iii) perimeter rantau berlorek.
the perimeter of the shaded region.
- (iv) luas rantau berlorek.
the area of the shaded region.

Faham

- (i) AB ialah perentas bagi sektor ACB , maka CP berserenjang dengan AB .
 AB is the chord for the sector ACB , hence CP is perpendicular to AB .

Diberi bahawa $CP = 5\sqrt{3}$ unit dan $AC = 10$ unit, AP boleh dihitung dengan menggunakan teorem Pythagoras.

Given that $CP = 5\sqrt{3}$ units and $AC = 10$ units. AP can be calculated by using Pythagoras theorem.

Seterusnya, diameter bulatan, $AB = 2AP$.
Hence, the diameter of semicircle, $AB = 2AP$.

- (ii) Sudut ACB dalam darjah dan minit boleh didapati dengan menggunakan $2 \times \cos \angle ACP$.
The angle ACB in degrees and minutes can be obtained by using $2 \times \cos \angle ACP$.

Kemudian, tukarkan ke radian.
Then, convert into radians

- (iii) Perimeter rantau berlorek
= panjang lengkok ADB + panjang lengkok AEB
Perimeter of the shaded region
= arc length ADB + arc length AEB

Tulis

- (i) $CP = 5\sqrt{3}$ unit/units
 $AC = 6 - (-4) = 10$ unit/units

$$AP = \sqrt{10^2 - (5\sqrt{3})^2} = 5 \text{ unit/units}$$

$$\text{Diameter } AB = 2(5) = 10 \text{ unit/units}$$

- (ii) $\cos/\cos \angle ACP = \frac{5\sqrt{3}}{10}$
 $\angle ACP = 30^\circ$
 $\angle ACB = 2(30^\circ) = 60^\circ$
 $60^\circ = \frac{60 \times n}{180} = \frac{\pi}{3}$
 $= \frac{\pi}{3} \text{ rad}$

- (iii) Panjang lengkok ADB / Arc length $ADB = \pi(5)$ unit
Panjang lengkok AEB / Arc length $AEB = \left(\frac{\pi}{3}\right)(10)$
Perimeter rantau berlorek
Perimeter of the shaded region
 $= 5\pi + \frac{10\pi}{3} = 8\frac{1}{3}\pi$ unit/units
 $= 26.18$ unit/units

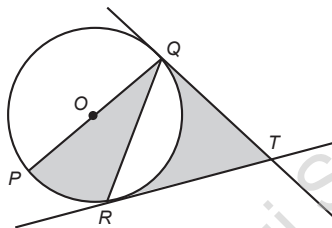
(iv) Luas rantau berlorek
 = luas semibulatan – luas tembereng
 Area of the shaded region
 = area of semicircle – area of segment

(iv) Luas tembereng AEB
 = luas sektor ACB – luas segi tiga ACB
 Area of segment AEB
 = area of sector ACB – area of the triangle ACB
 $= \frac{1}{2}(10)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(10)^2(\sin 60^\circ)$
 $= 52.37 - 43.30$
 $= 9.07 \text{ cm}^2$

Luas semibulatan ADB / Area of semicircle ADB
 $= \frac{1}{2}\pi(5)^2 = 39.28 \text{ unit}^2$

Luas rantau berlorek / Area of the shaded region
 $= 39.28 - 9.07 = 30.21 \text{ cm}^2$

CUBA DIRI



Rajah menunjukkan sebuah bulatan berpusat O dan jejari 12 cm. QT dan RT ialah tangen kepada bulatan itu masing-masing pada titik Q dan R. Diberi bahawa PQ ialah diameter dan $QR = 20$ cm. Cari
 The diagram shows a circle with centre O and radius 12 cm. QT and RT are tangents to the circle at the points of Q and R respectively. Given that PQ is the diameter and $QR = 20$ cm. Find

- (i) sudut QOR, dalam radian.
the angle QOR, in radians.
- (ii) perimeter, dalam cm, untuk keseluruhan rajah TQPR.
perimeter, in cm, of the whole diagram TQPR.
- (iii) jumlah luas, dalam cm^2 , rantau berlorek.
the total area, in cm^2 , of the shaded region.

Jawapan/Answer:
 (i) 1.97 rad
 (ii) 87.94 cm
 (iii) 225.81 cm^2

Taburan Kebarangkalian Probability Distribution

5.1 Pemboleh Ubah Rawak Random Variable

Buku Teks ms. 142 – 151

NOTA EKPRES

- Pemboleh ubah rawak ialah suatu pemboleh ubah dengan nilainya ialah kesudahan numerik yang dapat ditentukan daripada suatu fenomena rawak.
A random variable is a variable whose value is a numeric outcome from a random phenomenon.
- Ruang sampel mengandungi semua kejadian yang mungkin.
A sample space contains all the possible outcomes.
- Terdapat dua jenis pemboleh ubah rawak, iaitu pemboleh ubah rawak diskret dan pemboleh ubah rawak selanjar.
There are two types of random variables, namely discrete random variables and continuous random variables.
- Pemboleh ubah rawak diskret adalah ditulis dalam bentuk tatatanda set, $X = \{r : r = 0, 1, 2, 3\}$ manakala pemboleh ubah rawak selanjar ditulis dalam bentuk tatatanda set, $Y = \{y : y \text{ ialah isi padu air dalam liter, } a < y < b\}$.
Discrete random variable is written in set notation, $X = \{r : r = 0, 1, 2, 3\}$ while the continuous random variable is written in set notation, $Y = \{y : y \text{ is the volume of water in litre, } a < y < b\}$.

INFO



Taburan kebarangkalian pemboleh ubah rawak diskret
Probability distribution for discrete random variable

TUTORIAL



Pemboleh ubah rawak & tatatanda set
Random variable & set notation

- Lengkapkan yang berikut mengenai pemboleh ubah rawak.

SP
5.1.1

TP1

i-Think
Peta Pokok

Pemboleh ubah rawak Random variables

(a) **Pemboleh ubah rawak diskret**
Discrete random variable

(b) **Pemboleh ubah rawak selanjar**
Continuous random variable

(c) Pemboleh ubah rawak diskret ialah suatu pemboleh ubah yang dapat dikira. Jika kesudahan yang mungkin, X ialah 0, 1, 2, 3, 4 dan 5, maka ditulis dalam tatatanda set $X = \{x : x = 0, 1, 2, 3, 4, 5\}$.

A discrete random variable is a variable which can be counted. If the possible outcomes of X are 0, 1, 2, 3, 4 and 5, thus, it is written in set notation, $X = \{x : x = 0, 1, 2, 3, 4, 5\}$.

(d) Pemboleh ubah rawak selanjar ialah suatu pemboleh ubah Y yang nilainya tidak dapat dikira dan jika diambil nilai dari a ke b , maka ditulis dalam bentuk tatatanda set $Y = \{y : a \leq y \leq b\}$,

dengan keadaan a dan b ialah pemalar dan a kurang dan bukan sama dengan b .

A continuous random variable is a variable Y whose value cannot be counted and if it takes values from a to b , thus, it is written in set notation, $Y = \{y : a \leq y \leq b\}$, where a and b are constants and a is smaller and not equal to b .

2. Nyatakan pemboleh ubah rawak bagi setiap situasi yang berikut dan nyatakan semua kejadian yang mungkin dan tatatanda set sama ada ialah diskret atau selanjar. TP1

SP 5.1.2

State the random variable in each of the following situations and state all the possible outcomes in set notation and whether the random variable is discrete or continuous.

Contoh

- (i) Sekeping syiling dilambungkan sekali.
A coin is tossed once.
- (ii) Jisim sebiji nanas adalah di antara 0.3 kg dan 1.8 kg.
The mass of a pineapple is between 0.3 kg and 1.8 kg.
- (i) Pemboleh ubah ialah simbol di permukaan atas syiling, iaitu { kepala, bunga}. Diskret
The variable is the symbol on the top surface of a coin, which is { head, tail } Discrete
- (ii) Pemboleh ubah ialah jisim nanas, iaitu $\{ 0.3 \text{ kg} \leq \text{jisim} \leq 1.8 \text{ kg} \}$. Selanjar
The variable is the mass of a pineapple, which is $\{ 0.3 \text{ kg} \leq \text{mass} \leq 1.8 \text{ kg} \}$. Continuous

<p>(a) Nilai A dalam nombor $26A5_9$. <i>The value of A in the number $26A5_9$.</i></p> <p>Nilai A yang mungkin ialah 0, 1, 2, 3, 4, 5, 6, 7 dan 8. Diskret <i>The possible value of A is 0, 1, 2, 3, 4, 5, 6, 7 and 8. Discrete</i></p>	<p>(b) Tinggi sebatang tiang elektrik dipilih secara rawak ialah di antara 3.0 m ke 3.9 m. <i>The height of an electric pole which is chosen randomly is from 3.0 m to 3.9 m</i></p> <p>$\{ 3.0 \text{ m} \leq \text{tinggi tiang} \leq 3.9 \text{ m} \}$ Selanjar <i>$\{ 3.0 \text{ m} \leq \text{height of pole} \leq 3.9 \text{ m} \}$ Continuous</i></p>
<p>(c) Hasil tolak nombor pada dua dadu dilambungkan secara serentak. <i>The difference between the numbers on the two dice are tossed simultaneously.</i></p> <p>Pemboleh ubah ialah $\{0, 1, 2, 3, 4, 5\}$. Diskret <i>The variables are $\{0, 1, 2, 3, 4, 5\}$. Discrete</i></p>	<p>(d) Kedudukan keputusan seorang murid dalam kelas yang terdiri daripada 25 orang murid. <i>The result position of a pupil in a class which has 25 pupils.</i></p> <p>Pemboleh ubah kedudukan keputusan ialah $\{1, 2, 3, \dots, 25\}$ Diskret <i>The variables of the result position is $\{1, 2, 3, \dots, 25\}$ Discrete</i></p>

3. Lukis gambar rajah pokok untuk mewakili semua kesudahan X yang mungkin bagi yang berikut. Jadualkan taburan kebarangkalian bagi X. TP4

SP 5.1.3

Draw the tree diagrams to represent all the possible outcomes of X for each of the following. Tabulate the probability distribution for X.

Contoh

Dua dadu dilambungkan dua kali secara serentak dan X ialah pemboleh ubah diskret yang mewakili bilangan kali mendapat hasil tambah 5.
Two dice are tossed twice simultaneously and X is the discrete variable that represents the number of times of getting a sum of 5.

Tip Penting

Kebarangkalian peristiwa A berlaku $P(A) = \frac{n(A)}{n(S)}$, dengan keadaan $n(A)$ ialah bilangan peristiwa bagi A dan $n(S)$ ialah bilangan kesudahan dalam ruang sampel.

Probability of an event A happening $P(A) = \frac{n(A)}{n(S)}$, such that $n(A)$ is the number of event of A and $n(S)$ is the total outcomes in a sample space.

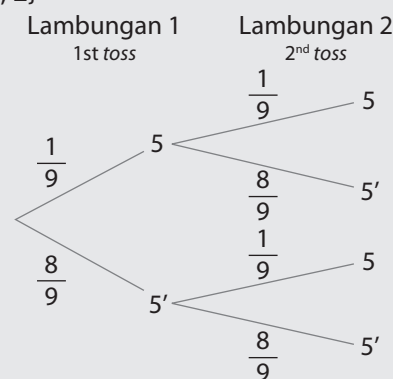
AKTIVITI PAK-21



Penerokaan SP 5.1.2
Exploring SP 5.1.2

Jawapan/ Answer

$X = \{ 0, 1, 2 \}$

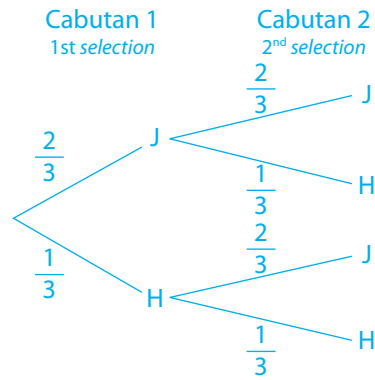


$X = r$	0	1	2
$P(X = r)$	$\left(\frac{8}{9}\right)\left(\frac{8}{9}\right) = \frac{64}{81}$	$2\left(\frac{1}{9}\right)\left(\frac{8}{9}\right) = \frac{64}{81}$	$\left(\frac{1}{9}\right)\left(\frac{1}{9}\right) = \frac{1}{81}$

- (a) Sebuah kotak mengandungi 2 keping kad hari jadi dan 4 keping kad jemputan. Sekeping kad dikeluarkan secara rawak dan jenisnya dicatatkan dan kemudian dikembali ke dalam kotak sebelum satu kad dikeluarkan lagi. X mewakili bilangan kali mendapat kad jemputan dalam dua cabutan berturut-turut.

A box contains 2 birthday cards and 4 invitation cards. A card is drawn at random from the box and the type of card is noted and it is then returned into the box before another card is taken out. X represents the number of times of drawing an invitation card when the process is repeated twice..

$$X = \{0, 1, 2\}$$



$X = r$	0	1	2
$P(X = r)$	$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$	$2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$	$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$

4. Pemboleh ubah rawak X mempunyai taburan kebarangkalian yang berikut.

TP 5

SP The random variable X has the following probability distribution.

- 5.1.4 (i) Bina jadual taburan kebarangkalian bagi X , dan tunjukkan X ialah satu pemboleh ubah rawak diskret
Construct a probability distribution table for X , and show that X is a discrete random variable.
- (ii) Lukis satu graf untuk taburan kebarangkalian X .
Draw a graph for the probability distribution of X .

Contoh

Diberi/ Given $P(X = x) = \frac{x}{6}$ untuk/for $x = \{0, 1, 2, 3\}$

$$P(X = 0) = \frac{0}{6} = 0$$

$$P(X = 1) = \frac{1}{6}$$

$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$

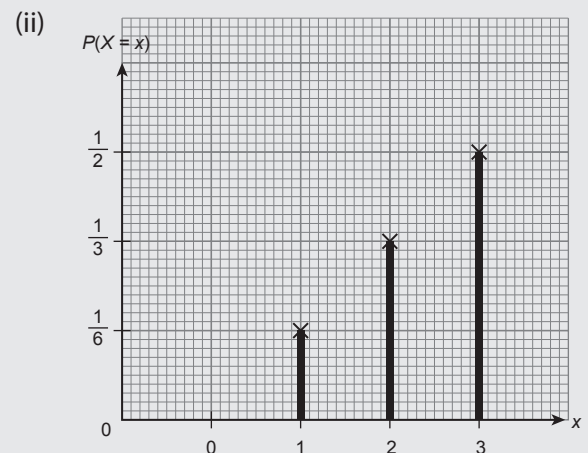
$$P(X = 3) = \frac{3}{6} = \frac{1}{2}$$

$X = x$	0	1	2	3
$P(X = x)$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

$$\begin{aligned} P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\ = 0 + \frac{1}{6} + \frac{1}{3} + \frac{1}{2} \\ = 1 \text{ (Terbukti)} \end{aligned}$$

Tip Penting

Untuk menunjukkan X ialah satu pemboleh ubah rawak diskret, kita mesti pastikan $\sum_0^n P(X = x) = 1$.
To show that X is a discrete random variable, we must show that $\sum_0^n P(X = x) = 1$.



(a) Diberi/ Given $P(X = x) =$ untuk/ for $x = \{2, 4, 6, 8\}$.

$$P(X=2) = \frac{2(2) - 1}{36} = \frac{1}{12}$$

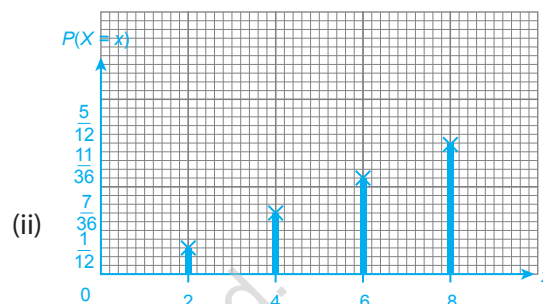
$$P(X=4) = \frac{2(4) - 1}{36} = \frac{7}{36}$$

$$P(X=6) = \frac{2(6) - 1}{36} = \frac{11}{36}$$

$$P(X=8) = \frac{2(8) - 1}{36} = \frac{5}{12}$$

$X = x$	2	4	6	8
$P(X = x)$	$\frac{1}{12}$	$\frac{7}{36}$	$\frac{11}{36}$	$\frac{5}{12}$

$$\begin{aligned} P(X=2) + P(X=4) + P(X=6) + P(X=8) \\ = \frac{1}{12} + \frac{7}{36} + \frac{11}{36} + \frac{5}{12} \\ = 1 \text{ (Terbukti)} \end{aligned}$$



5. Selesaikan setiap soalan yang berikut.

TP 5

SP Solve each of the following questions.
5.1.4

Contoh

X ialah suatu pemboleh ubah rawak diskret dengan taburan kebarangkalian yang berikut.
 X is a discrete random variable with the following probability distribution.

X	0	1	2	3
$P(X = x)$	0.14	0.32	0.24	0.30

Cari/Find (i) $P(X < 1)$ (ii) $P(0 \leq X < 2)$

$$\begin{aligned} \text{(i) } P(X < 1) &= P(X = 0) \\ &= 0.14 \end{aligned}$$

$$\begin{aligned} \text{(ii) } P(0 \leq X < 2) &= P(x = 0) + P(X = 1) \\ &= 0.14 + 0.32 \\ &= 0.46 \end{aligned}$$

(a) X ialah suatu pemboleh ubah rawak diskret dengan taburan kebarangkalian yang berikut.
 X is a discrete random variable with the following probability distribution.

X	1	3	5	7
$P(X = x)$	0.17	0.23	0.36	p

Cari/Find (i) nilai p .
the value of p .
(ii) $P(X < 7)$

(iii) $P(1 \leq X < 5)$

$$\begin{aligned} \text{(i) } p &= 1 - (0.17 + 0.23 + 0.36) = 0.24 \\ \text{(ii) } P(X < 7) &= 1 - P(X = 7) \\ &= 1 - 0.24 = 0.76 \end{aligned}$$

$$\begin{aligned} \text{(iii) } P(1 \leq X < 5) &= 1 \\ &= P(X = 1) + P(X = 3) \\ &= 0.17 + 0.23 = 0.40 \end{aligned}$$

5.2 Taburan Binomial Binomial Distribution

NOTA EKSPRES

1. Suatu eksperimen dengan dua kesudahan sahaja dikenali sebagai eksperimen Bernoulli.
An experiment with only two outcomes is known as Bernoulli experiment.
2. Dua kesudahan adalah dipanggil kejayaan dan kegagalan. Jika kebarangkalian kejayaan ialah p , maka kebarangkalian kegagalan q ialah $1 - p$.
The two outcomes are commonly known as success and failure. If the probability of success is p , then the probability of failure q is $1 - p$.
3. Jika eksperimen Bernoulli diulangi n kali secara tak bersandar, maka eksperimen tersebut dikenali Eksperimen Binomial.
When the Bernoulli experiment is repeated n times independently, then the experiment is known as Binomial experiment.
4. Fungsi kebarangkalian binomial bagi X diberi oleh rumus:
The binomial probability function for X is given by the formula:
 $P(X = r) = {}^n C_r p^r q^{n-r}, r = 1, 2, 3, \dots, n$
5. Notasi bagi pemboleh ubah rawak diskret X yang bertaburan binomial dengan n percubaan dan kebarangkalian kejayaan p ditulis secara $X \sim B(n, p)$
The notation of a discrete random variable X which is binomial distributed with n number of trials and the probability of each success p can be written as $X \sim B(n, p)$

6. Selesaikan yang berikut.

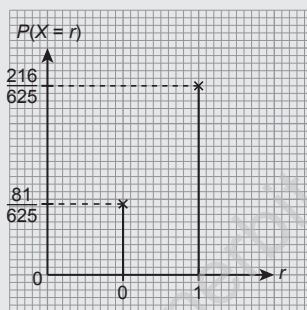
TP 6

SP
5.2.1 Solve the following.

Contoh

Rajah menunjukkan graf $X \sim B(n, 0.4)$ yang tidak lengkap.

The diagram shows the graph $X \sim B(n, 0.4)$ that is incomplete.



- (i) Cari nilai n / Find the value of n .
- (ii) Seterusnya, cari $P(X > 3)$. / Then, find $P(X > 3)$.

(i) $X \sim B(n, 0.4)$

$$P(X = 0) = {}^n C_0 (0.4)^0 (0.6)^n = \frac{81}{625}$$

$$\left(\frac{3}{5}\right)^n = \frac{81}{625} = \left(\frac{3}{5}\right)^4$$

$$n = 4$$

$$\begin{aligned} \text{(ii) } P(X > 3) &= P(X = 4) \\ &= {}^4 C_4 (0.4)^4 \\ &= \frac{16}{625} \end{aligned}$$

- (a) Diberi X ialah suatu pemboleh ubah rawak diskret dengan keadaan $X \sim B(6, p)$ dan $P(X = 6) = \frac{1}{64}$. Cari nilai p .

Given that X is a discrete random variable, such that $X \sim B(6, p)$ and $P(X = 6) = \frac{1}{64}$. Find the value of p .

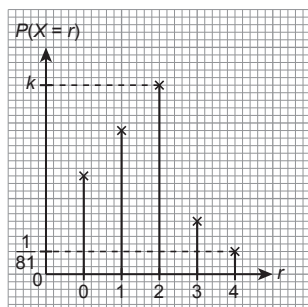
$$X \sim B(6, p)$$

$$P(X = 6) = {}^6 C_6 p^6 = \frac{1}{64}$$

$$p^6 = \left(\frac{1}{2}\right)^6$$

$$p = \frac{1}{2}$$

(b) Rajah menunjukkan graf taburan binomial
The diagram shows a binomial distribution graph X



- (i) Cari nilai k and nilai p .
Find the value of k and of p .
- (ii) Tunjukkan bahawa $P(X=0) = 2P(X=3)$
Show that $P(X=0) = 2P(X=3)$

(i) $X \sim B(4, p)$

$$P[X=4] = {}^4C_4 p^4 = \frac{1}{81}$$

$$p^4 = \left(\frac{1}{3}\right)^4$$

$$p = \frac{1}{3}$$

$$P(X=2) = {}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2$$

$$= \frac{24}{81} = \frac{8}{27}$$

$$\therefore k = \frac{8}{27}, p = \frac{1}{3}$$

(ii) $P[X=0] = {}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

$$P[X=3] = {}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 = \frac{8}{81}$$

$$P(X=0) = 2 \left(\frac{8}{81}\right)$$

$$\therefore P(X=0) = 2P(X=3)$$

(Tertunjuk / Shown)

(c) Taburan kebarangkalian bagi satu pemboleh ubah rawak diskrit $X = \{0, 1, 2, 3, 4\}$ diberi oleh $P(X=r) = m(2r+1)$ bagi setiap nilai r .
The probability distribution for a discrete random variable $X = \{0, 1, 2, 3, 4\}$ is given by $P(X=r) = m(2r+1)$ for each value of r .

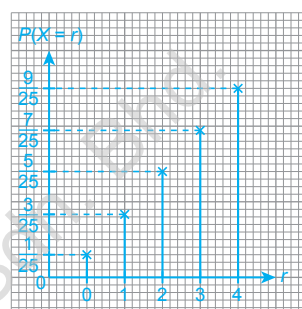
- (i) Cari nilai m .
Find the value of m
- (ii) Lukis satu graf bagi taburan kebarangkalian X
Draw a graph for the probability distribution of X

(i) $m(1) + 3m + 5m + 7m + 9m = 1$
 $25m = 1$

$$m = \frac{1}{25}$$

$$P(X=r) = m(2r+1)$$

(ii)



7. Tentukan kebarangkalian suatu peristiwa bagi taburan binomial yang berikut.

TP 4

SP 5.2.2 Determine the distribution of an event with the following binomial distribution.

Contoh

Dalam suatu kaji selidik, 80% orang yang tidak memakai pelitup muka didedahkan kepada orang yang mempunyai penyakit Covid-19 akan diuji positif. Jika 7 orang daripada kumpulan tersebut dipilih secara rawak, cari peratus bahawa

In a research, 80% of people without wearing face masks exposed to any Covid-19 patient will be tested positive. If 7 people are chosen at random from this group of people, find the percentage that

- (i) semua dijangkiti Covid-19.
all contracted the Covid-19 disease.
- (ii) lebih daripada 3 orang dijangkiti Covid-19.
more than 3 people contracted Covid-19 disease.

Katakan X ialah bilangan orang yang tidak memakai pelitup muka didedahkan kepada orang yang mempunyai penyakit Covid-19 akan diuji positif.

Let X be the number of people without wearing face masks exposed to any Covid-19 patient will be tested positive.

Maka/Thus $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$

$n = 7, p = 0.8, q = 0.2$

(i) $P(X = 7) = {}^7C_7(0.8)^7 = 0.2097$
Maka, 20.97% akan dijangkiti.
Thus, 20.97% is contracted.

(ii) $P(X > 3) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$
 $= {}^7C_4(0.8)^4(0.2)^3 + {}^7C_5(0.8)^5(0.2)^2 + {}^7C_6(0.8)^6(0.2)^1 + {}^7C_7(0.8)^7(0.2)^0$
 $= 0.9667$

Maka/Thus 96.67 %

KALKULATOR



(a) Pada suatu gerai, kebarangkalian seorang pelanggan membeli pisang goreng ialah 55%. Jika 9 orang pelanggan datang ke gerai tersebut, cari kebarangkalian

At a stall, the probability of a customer buying fried bananas is 55%. If 9 customers come to the stall, find the probability that

(i) tepat 4 pelanggan membeli pisang goreng.
exactly 4 customers buy fried bananas.

(ii) kurang daripada 3 pelanggan membeli pisang goreng.
less than 3 customers buy fried bananas.

$$n = 9, p = 0.55, q = 0.45$$

$$(i) P(X = 4) = {}^9C_4(0.55)^4(0.45)^5 = 0.2128$$

$$(ii) P(X < 3) = P(X = 0) - P(X = 1) + P(X = 2) = {}^9C_0(0.55)^0(0.45)^9 + {}^9C_1(0.55)^1(0.45)^8 + {}^9C_2(0.55)^2(0.45)^7 = 0.0498$$

(b) Kebarangkalian Ronny akan menang dalam pertandingan perbahasan ialah $\frac{5}{7}$. Jika dia mengambil bahagian dalam n pertandingan, kebarangkalian dia tewas dalam semua pertandingan ialah 0.0019. Cari

The probability that Ronny will win in the debate competition is $\frac{5}{7}$. If he takes part in n competitions, the probability that he loses all the competitions is 0.0019. Find

(i) nilai n .
the value of n .

(ii) kebarangkalian dia menang lebih daripada tiga kali dalam n pertandingan ini
the probability he will win more than three times out of the n competitions.

$$p = \frac{5}{7}, q = \frac{2}{7}$$

$$(i) P(X = 0) = 0.0019 = {}^nC_0 \left(\frac{5}{7}\right)^0 \left(\frac{2}{7}\right)^n = 0.0019$$

$$(ii) P(X > 3) = P(X = 4) + P(X = 5) = {}^5C_4 \left(\frac{5}{7}\right)^4 \left(\frac{2}{7}\right)^1 + {}^5C_5 \left(\frac{5}{7}\right)^5 \left(\frac{2}{7}\right)^0 = 0.5578$$

$$n \log \left(\frac{2}{7}\right) = \log 0.0019$$

$$n = \frac{\log 0.0019}{\log \left(\frac{2}{7}\right)} = 5$$

(c) Satu kajian menunjukkan bahawa 25% daripada murid di sebuah bandar berbasikal ke sekolah. Jika 9 orang murid dipilih secara rawak dari sekolah itu, cari kebarangkalian bahawa

A survey shows that 25% of the pupils from a town cycle to school. If 9 pupils are chosen at random from the school, find the probability that

(i) tepat 3 orang murid tidak berbasikal ke sekolah.
exactly 3 pupils do not cycle to school.

(ii) sekurang-kurangnya 2 orang murid berbasikal ke sekolah.
at least 2 pupils cycle to school.

$$(i) p = 0.75, q = 0.25$$

$$P(X = 3) = {}^9C_3(0.75)^3(0.25)^6 = 0.00865$$

$$(ii) p = 0.25, q = 0.75$$

$$P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y = 0) - P(Y = 1) = 1 - {}^9C_0(0.25)^0(0.75)^9 - {}^9C_1(0.25)(0.75)^8 = 0.6997$$

8. Selesaikan masalah yang berikut.

SP 5.2.3 Solve the following problems.

Contoh

Didapati bahawa kebarangkalian sebuah buku rujukan Matematik digemari oleh pelajar ialah 0.68. 5 buku rujukan Matematik dipilih secara rawak.

It is known that the probability of a Mathematics reference book that is preferred by the students is 0.68. 5 Mathematics reference books are chosen at random.

(i) Bina jadual taburan kebarangkalian binomial bagi bilangan buku rujukan Matematik digemari oleh pelajar.

Construct a binomial probability distribution table for the number of Mathematics reference books preferred by the students.

(ii) Daripada jadual (i), lukis graf taburan kebarangkalian tersebut.

From the table (i), draw a probability distribution graph.

(iii) Tentukan kebarangkalian lebih daripada 3 buah buku rujukan Matematik digemari oleh pelajar.

Determine the probability of more than 3 Mathematics reference books are preferred by the students

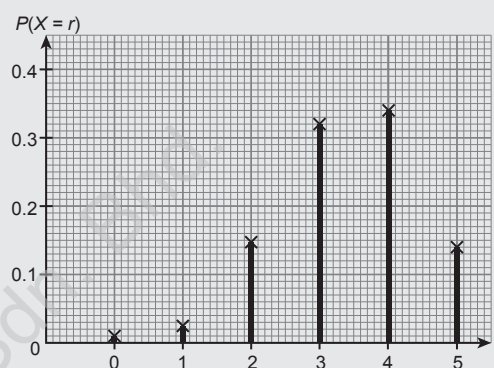
(i) Katakan X mewakili bilangan buku rujukan Matematik digemari oleh pelajar.

Let X be the number of Mathematics reference books preferred by the students.

$$X = \{0, 1, 2, 3, 4, 5\}$$

$$n = 5, p = 0.68, q = 0.32$$

$X = r$	$P(X = r)$
0	${}^5C_0(0.68)^0(0.32)^5 = 0.0034$
1	${}^5C_1(0.68)^1(0.32)^4 = 0.0357$
2	${}^5C_2(0.68)^2(0.32)^3 = 0.1515$
3	${}^5C_3(0.68)^3(0.32)^2 = 0.3220$
4	${}^5C_4(0.68)^4(0.32)^1 = 0.3421$
5	${}^5C_5(0.68)^5(0.32)^0 = 0.1454$



$$(iii) P(X > 3) = P(X = 4) + P(X = 5) = 0.3421 + 0.1454 = 0.4875$$

(a) Didapati bahawa kebarangkalian sebuah televisyen pembuatan Jepun dalam sebuah kedai elektrik ialah 0.64. Satu sampel terdiri daripada 4 buah televisyen dipilih secara rawak dari kedai itu.

It is known that the probability of a television made in Japan in an electrical shop is 0.64. A sample of 4 televisions is chosen randomly from the shop.

(i) Bina jadual taburan kebarangkalian binomial bagi bilangan televisyen pembuatan Jepun.

Construct a binomial probability distribution table for the number of televisions made in Japan.

(ii) Daripada jadual (i), lukis graf taburan kebarangkalian tersebut.

From the table (i), draw a probability distribution graph.

(iii) Tentukan kebarangkalian sekurang-kurangnya 2 televisyen pembuatan Jepun dalam kedai elektrik itu.

Determine the probability at least 2 televisions are made in Japan in the electrical shop.

(i) Katakan X mewakili bilangan televisyen pembuatan Jepun.

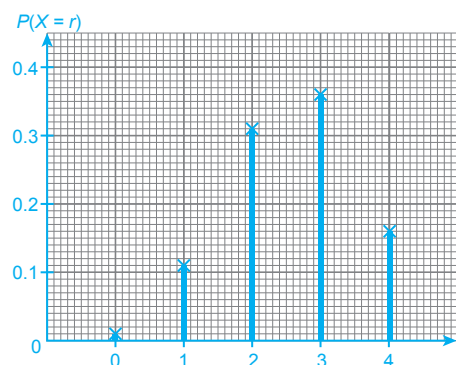
Let X be the number of televisions made in Japan.

$$X = \{0, 1, 2, 3, 4\}$$

$$n = 4, p = 0.64, q = 0.36$$

$X = r$	$P(X = r)$
0	${}^4C_0(0.64)^0(0.36)^4 = 0.0168$
1	${}^4C_1(0.64)^1(0.36)^3 = 0.1194$
2	${}^4C_2(0.64)^2(0.36)^2 = 0.3185$
3	${}^4C_3(0.64)^3(0.36)^1 = 0.3775$
4	${}^4C_4(0.64)^4(0.36)^0 = 0.1678$

(ii)



$$(iii) P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.0168 - 0.1194 = 0.8638$$

(b) Kebarangkalian tidak mendapat satu pun lampu merah daripada 5 percubaan ditunjukkan dalam graf yang diberi.

The probability of not getting any red bulb from 5 attempts is shown in the given graph.

(i) Tentukan nilai kebarangkalian p bagi mendapat sebiji lampu berwarna merah.
Determine the probability p of choosing a red bulb.

(ii) Bina jadual taburan kebarangkalian binomial bagi mendapat lampu merah bagi 5 percubaan itu, dan lengkapkan graf taburan kebarangkaliannya.

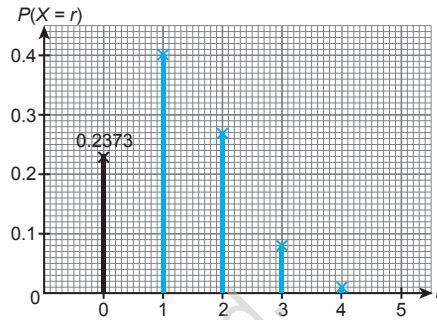
Construct a binomial probability distribution table to obtain red bulbs for the 5 attempts and complete the probability distribution graph.

$$(i) P(X = 0) = {}^5C_0(p^0)(1 - p)^5 = 0.2373$$

$$1 - p = 0.75$$

$$p = 0.25$$

(ii) $X = r$	$P(X = r)$
0	${}^5C_0(0.25)^0(0.75)^5 = 0.2373$
1	${}^5C_1(0.25)^1(0.75)^4 = 0.3955$
2	${}^5C_2(0.25)^2(0.75)^3 = 0.2637$
3	${}^5C_3(0.25)^3(0.75)^2 = 0.0879$
4	${}^5C_4(0.25)^4(0.75)^1 = 0.0146$
5	${}^5C_5(0.25)^5(0.75)^0 = 0.0010$

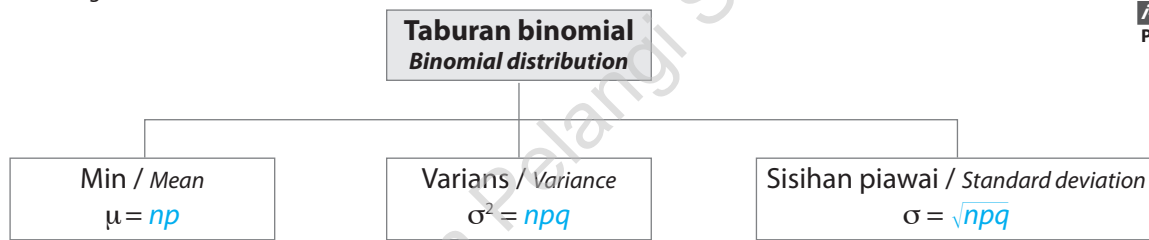


9. Lengkapkan rajah dengan rumus taburan binomial.

SP Complete the diagram with the formulae related to binomial distribution.
5.2.4

TP 3

i-Think
Peta Pokok



10. Selesaikan masalah yang melibatkan nilai min, varians dan sisihan piawai bagi suatu taburan binomial.

SP Solve the following problems involving mean, variance and standard deviation of binomial distributions.
5.2.4

TP 4

Contoh

$\frac{1}{15}$ daripada murid dalam Darjah 6 memperoleh cemerlang dalam ujian kelayakan. Cari min dan sisihan piawai bilangan murid dalam darjah 6 memperoleh cemerlang daripada 255 orang murid.

$\frac{1}{15}$ of the pupils in Standard 6 obtained distinction in the qualifying test. Find the mean and standard deviation of the number of pupils in Standard 6 who obtained distinction in the qualifying test if there are 255 pupils.

Katakan X mewakili bilangan murid dalam Darjah 6 memperoleh cemerlang dalam ujian kelayakan.

Let X be the number of pupils in Standard 6 who get distinction in the qualifying test.

$$n = 255, p = \frac{1}{15}, q = \frac{14}{15}$$

$$\text{min/mean} = E(X) = np = 255 \times \frac{1}{15} = 17$$

$$\text{sisihan piawai (X)} = \sqrt{255 \left(\frac{1}{15}\right) \left(\frac{14}{15}\right)} = 3.98$$

standard deviation (X)

(a) Terdapat x buah nanas daripada y buah nanas dari gerai itu ialah manis. Jika n bilangan nanas dipilih secara rawak daripada gerai itu.

There is x out of y pineapples from the stall are sweet. If n is the number of pineapples chosen randomly from the stall,

(i) ungkapkan min bilangan nanas dalam sebutan x, y dan n .

express the mean number of pineapples in terms of x, y and n .

(ii) cari nilai x jika $\mu = 10, \sigma = 2.5$ dan $y = 40$.
find the value of x if $\mu = 10, \sigma = 2.5$ and $y = 40$.

$$(i) \text{Min/Mean} = np = n \left(\frac{x}{y}\right)$$

$$(ii) 10 = np$$

$$2.5^2 = npq$$

$$q = \frac{2.5^2}{10} = 0.625$$

$$p = 0.375$$

$$\frac{x}{y} = 0.375$$

$$x = 0.375 \times 40 = 15$$

(b) Satu pemboleh ubah rawak diskret X bertaburan binomial $B(n, 0.56)$. Jika min ialah 28, cari
A discrete random variable X has a binomial distribution $B(n, 0.56)$. If the mean is 28, find

- (i) nilai n .
the value of n .
 (ii) varians dan sisihan piawai.
the variance and the standard deviation.

$$\begin{aligned} \text{(i) } E(X) &= np = 28 \\ n(0.56) &= 28 \\ n &= 50 \end{aligned}$$

$$\begin{aligned} \text{(ii) Varians/Variance} &= npq \\ &= 50 \times 0.56 \times 0.44 \\ &= 12.32 \end{aligned}$$

$$\text{Sisihan piawai/Standard deviation} = 3.51$$

(c) X bertaburan binomial dengan $B(n, p)$. Jika q ialah 0.28 dan sisihan piawai = 2.42, cari
 X is binomial distributed with $B(n, p)$. If q is 0.28 and the standard deviation = 2.4, find

- (i) nilai n .
the value of n .
 (ii) min/mean

$$\begin{aligned} \text{(i) } q &= 0.28, p = 0.72 \\ \text{Sisihan piawai/Standard deviation} &= 2.42 = \sqrt{n(0.72)(0.28)} \\ n &= \frac{2.42^2}{[(0.72)(0.28)]} \\ &= 29 \end{aligned}$$

$$\begin{aligned} \text{(ii) Min/Mean} &= np = 29(0.72) \\ &= 20.88 \end{aligned}$$

11. Selesaikan setiap yang berikut.

SP Solve each of the following.
 5.2.5

TP 5

Contoh

2% daripada pesakit adalah alergik kepada ubat antibiotik A. Cari bilangan pesakit perlu disemak supaya kebarangkalian bahawa sekurang-kurangnya seorang pesakit adalah alergik ialah lebih daripada 0.95.

2% of the patients are allergic to an antibiotic A. Find the number of patients that must be inspected so that the probability that at least one of them is allergic to the antibiotic is more than 0.95.

$$\begin{aligned} p &= 0.02, q = 0.98 \\ P(X \geq 1) &> 0.95 \\ 1 - P(X = 0) &> 0.95 \\ 1 - {}^n C_0 (0.02)^0 (0.98)^n &> 0.95 \\ 0.98^n &< 0.05 \\ n \log 0.98 &< \log 0.05 \\ n &> \frac{\log 0.05}{\log 0.98} \\ n &> 148.28 \\ n &= 149 \end{aligned}$$

Tip Penting

$$\begin{aligned} n \log 0.98 &< \log 0.05 \\ n(-0.0088) &< -1.3010 \\ n &> \frac{1.301}{0.0088} \\ n &> 148.28 \\ n &= 149 \end{aligned}$$

(a) 40% daripada remaja membeli keperluannya dalam talian. Didapati bahawa jika satu sampel terdiri daripada n orang remaja, kebarangkalian tidak ada seorang pun membeli keperluannya dalam talian adalah 7.6 kali daripada kebarangkalian semua orang yang membeli keperluan dalam talian. Cari

40% of the youth buy their needs online. It is found from a sample of n youths, the probability that none buys his/her needs online is 7.6 times the probability that all of them buy online. Find

- (i) nilai n .
the value of n .
 (ii) min dan sisihan piawai dengan n tersebut.
the mean and standard deviation with this n .

$$\begin{aligned} \text{(i) } p &= 0.4, q = 0.6 \\ P(X = 0) &= 7.6 P(X = n) \\ {}^n C_0 (0.4)^0 (0.6)^n &= 7.6 {}^n C_n (0.4)^n \\ \left(\frac{0.6}{0.4}\right)^n &= 7.6 \\ n \log 1.5 &= \log 7.6 \\ n &= \frac{\log 7.6}{\log 1.5} \\ n &= 5 \end{aligned}$$

$$\begin{aligned} \text{(ii) Min/Mean} &= np = 5(0.4) = 2 \\ \text{Sisihan piawai/Standard deviation} &= npq \\ &= [2(0.6)]^{\frac{1}{2}} = 1.1 \end{aligned}$$

(b) Dalam suatu tinjauan Kementerian Kesihatan, didapati bahawa 30% daripada rakyat Malaysia takut mengambil vaksin COVID-19.

In a survey done by the Health Ministry, it is found that 30% of Malaysians are afraid to take Covid-19 vaccines.

(i) Jika 10 orang dipilih secara rawak, cari kebarangkalian bahawa lebih daripada 50% takut mengambil vaksin tersebut.

If 10 Malaysians are chosen at random, find the probability that more than 50% of them are afraid to take the vaccines.

(ii) Cari min bilangan rakyat tidak takut mengambil vaksin tersebut daripada sebuah pekan yang terdiri daripada 7 500 penduduk.

Find the mean number of people who are not afraid to take the vaccine from a town of 7 500 residents.

$$\begin{aligned} \text{(i)} \quad P(X > 5) &= {}^{10}C_6(0.3)^6(0.7)^4 + {}^{10}C_7(0.3)^7(0.7)^3 + \\ & {}^{10}C_8(0.3)^8(0.7)^2 + {}^{10}C_9(0.3)^9(0.7)^1 + \\ & {}^{10}C_{10}(0.3)^{10}(0.7)^0 \\ &= 0.0473 \end{aligned}$$

$$\text{(ii)} \quad E(X) = np = 7500 \times 0.7 = 5250$$

(c) Kebarangkalian pelanggan yang membuat tempahan sebuah hotel tidak hadir ialah 0.45. Hotel tersebut mempunyai 150 bilik sewa, cari kebarangkalian bahawa

The probability that customers who made bookings at a hotel do not turn up is 0.45. The hotel has 150 rooms, find the probability that

(i) tepat 10 pelanggan hadir daripada 12 tempahan.

exactly 10 customers come from 12 reservations.

(ii) cari min dan sisihan piawai jika 15 ditempahkan.

find the mean and standard deviation if 15 rooms are booked.

$$n = 12, p = 0.45, q = 0.55$$

$$\text{(i)} \quad P(X = 10) = {}^{12}C_{10}(0.45)^{10}(0.55)^2 = 1.03 \times 10^{-4}$$

$$\text{(ii)} \quad E(X) = np = 15 \times 0.45$$

$$= 6.75$$

$$\sigma = \sqrt{6.75(0.55)} = 1.93$$

(d) Dalam lambungan sebiji dadu adil, lambungan yang menghasilkan nombor perdana dianggap sebagai kejayaan. Cari bilangan lambungan minimum yang perlu dilakukan supaya kebarangkalian mendapat sekurang-kurangnya satu kejayaan adalah lebih besar daripada 0.9.

In a tossing of dice, getting a prime number is considered a success. Find the minimum number of times to toss the dice in order for the probability success rate to be more than 0.9.

$$p = \frac{1}{2}, q = \frac{1}{2}$$

$$P(X \geq 1) > 0.9$$

$$1 - P(X = 0) > 0.9$$

$$P(X = 0) < 0.1$$

$${}^nC_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n < 0.1$$

$$n \log \left(\frac{1}{2}\right) < \log 0.1$$

$$n > 3.3$$

$$n = 4$$

(e) X ialah pemboleh ubah rawak diskrit dengan bilangan percubaan sebanyak 12 kali dan sisihan piawai ialah $\sqrt{\frac{5}{3}}$. Jika p ialah kebarangkalian bagi kejayaan,

X is a discrete random variable with 12 trials and the standard deviation of $\sqrt{\frac{5}{3}}$. If p is the probability of success,

(i) cari nilai-nilai yang mungkin bagi p .

find the possible values of p .

(ii) Seterusnya, gunakan nilai p yang lebih besar daripada bahagian (i), cari kebarangkalian mendapat kejayaan lebih daripada 10 kali.

Then, using the bigger value of p from part (i), find the probability of success of more than 10 times.

$$\text{(i)} \quad \left(\sqrt{\frac{5}{3}}\right)^2 = npq$$

$$\frac{5}{3} = 12p(1 - p)$$

$$36p^2 - 36p + 5 = 0$$

$$(6p - 5)(6p - 1) = 0$$

$$p = \frac{5}{6} \text{ atau/or } p = \frac{1}{6}$$

$$\text{(ii)} \quad P(X > 10) = P(X = 11) + P(X = 12)$$

$$= {}^{12}C_{11} \left(\frac{5}{6}\right)^{11} \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^{12}$$

$$= 0.269176 + 0.112157$$

$$= 0.3813$$

5.3 Taburan Normal Normal Distribution

NOTA EKSPRES

- Taburan normal** bagi suatu pemboleh ubah rawak selanjur X dengan min μ dan sisihan piawai σ boleh ditukar kepada **taburan normal piawai** melalui rumus $Z = \frac{X - \mu}{\sigma}$, dengan keadaan Z dinamakan skor piawai atau skor-z.

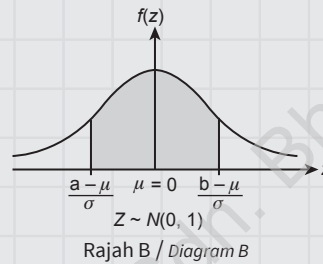
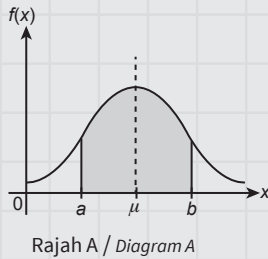
The **normal distribution** can be converted into **standard normal distribution** by using the formula $Z = \frac{X - \mu}{\sigma}$, such that Z is called the standard score or z-score.

- Pemboleh ubah rawak normal piawai, Z mempunyai nilai min $\mu = 0$ dan sisihan piawai $\sigma = 1$. Z dikatakan bertaburan normal piawai, iaitu $Z \sim N(0, 1)$.

The standard normal random variable, Z has a mean value of $\mu = 0$ and the standard deviation $\sigma = 1$. Z is standard normal distributed, which is denoted by $Z \sim N(0, 1)$.

- Rajah A menunjukkan taburan normal bagi pemboleh ubah $X \sim N(\mu, \sigma^2)$, dengan min μ dan sisihan piawai σ . Rajah B menunjukkan lakaran taburan normal piawai $Z \sim N(0, 1)$.

Diagram A shows the normal distribution for variable $X \sim N(\mu, \sigma^2)$, with mean μ and standard deviation σ . Diagram B shows the standard normal distribution for $Z \sim N(0, 1)$.



TUTORIAL



Taburan normal
Normal distribution

- Apabila luas berlorek dalam Rajah A ialah $P(a < X < b)$, maka luas dalam Rajah B ialah $P\left(\frac{a - \mu}{\sigma} < Z < \frac{b - \mu}{\sigma}\right)$.

When the shaded area in Diagram A is $P(a < X < b)$, then the area in Diagram B is $P\left(\frac{a - \mu}{\sigma} < Z < \frac{b - \mu}{\sigma}\right)$.

- Rajah menunjukkan suatu graf fungsi taburan normal bagi suatu pemboleh ubah rawak selanjur x dengan min μ dan sisihan piawai σ . **TP 2**

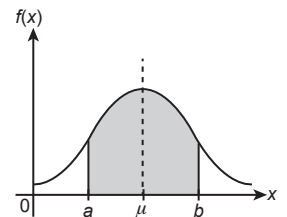
SP

5.3.1

Diagram shows a normal distribution function graph for a continuous random variable X with mean μ and standard deviation σ .

Lengkapkan rajah berikut mengenai ciri-ciri graf fungsi taburan normal.

Complete the following diagram about the features of the normal distribution function graph.



i-Think
Peta Buih

Berbentuk loceng dan bersimetri
pada min μ .
It is bell-shaped and symmetrical about the
mean μ .

Tatatanda bagi luas
berlorek dari $x = a$ ke $x = b$ ditulis
sebagai $P(a \leq x \leq b)$.
The notation of the shaded
area from $x = a$ to $x = b$ is written
as $P(a \leq x \leq b)$.

**Graf fungsi taburan
normal**
**Normal distribution
function graph**

Jumlah luas di antara graf dan
paksi-x bernilai 1.
The total area under the curve and the
x-axis is 1.

Tatatanda taburan normal
bagi pemboleh ubah X ditulis sebagai
 $X \sim N(\mu, \sigma^2)$.
The notation of the normal distribution
for variable X is written as
 $X \sim N(\mu, \sigma^2)$.

13. Selesaikan yang berikut. **TP 3**

SP Solve the following.
5.3.2

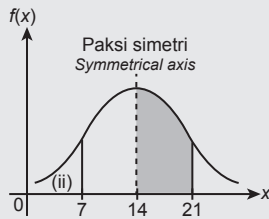
VIDEO



Jadual taburan normal piawai, skor-z
Standard normal distribution table, z-scores.

Contoh

Rajah menunjukkan graf fungsi taburan normal bagi suatu pemboleh ubah rawak selangar X .
Diagram shows the normal distribution function graph of a continuous random variable X .



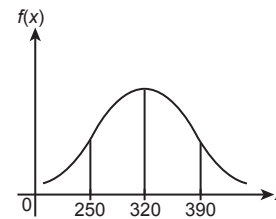
- (i) Nyatakan nilai μ .
State the value of μ .
- (ii) Lorekkan rantau $P(X < 7)$.
Shade the region $P(X < 7)$.
- (iii) Ungkapkan rantau berlorek dalam tatatanda kebarangkalian
Express the shaded region in probability notation.
- (iv) Cari/Find $P(X > 14)$

Tip Penting

Graf bersimetri pada min μ .
The graph is symmetrical about the mean μ .

- (i) $\mu = 4$
- (ii) Dilorek pada rajah / Shaded on the graph
- (iii) $P(14 < X < 21)$
- (iv) $P(X > 14) = 0.5$

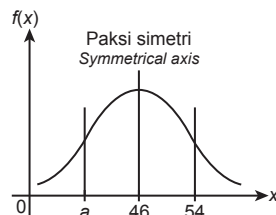
- (a) Rajah menunjukkan graf taburan normal bagi suatu pemboleh ubah rawak selangar X .
Diagram shows the normal distribution of a continuous random variable X .



- (i) Nyatakan nilai μ .
State the value of μ .
- (ii) Nyatakan nilai $P(X > \mu)$.
State the value of $P(X > \mu)$.
- (iii) Cari/Find $P(250 < X < 390)$ jika/if $P(X < 250) = 0.24$.

- (i) $\mu = 320$
- (ii) $P(X > 320) = 0.5$
- (iii) $P(250 < X < 390) = 1 - 2(0.24) = 0.52$

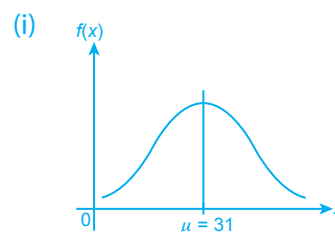
- (b) Rajah menunjukkan graf taburan normal bagi suatu pemboleh ubah rawak selangar X .
Diagram shows the normal distribution of a continuous random variable X



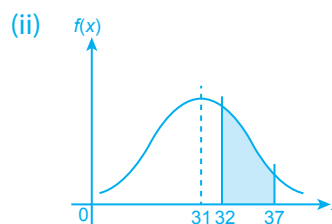
- (i) Jika/If $P(X < a) = P(X > 54)$, cari/find a .
- (ii) Jika/If $P(a < X < 54) = 0.86$ dan/and $P(X > 54) = 0.07$, cari/find $P(X < a)$
- (iii) Jika/If $P(a < X \leq 46) = 0.231$, cari/find $P(X > a)$

- (c) Suatu pemboleh ubah rawak selangar X bertaburan normal. Lakar satu taburan normal untuk menunjukkan yang berikut:
A continuous random variable X has a normal distribution. Sketch the normal graph to show the following:

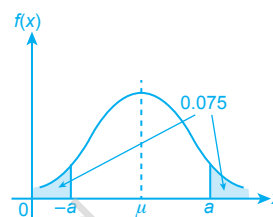
- (i) min/mean $\mu = 31$
- (ii) lorek/shade $P(32 < X < 37)$
- (iii) lorek $P(|X| > a) = 0.15$ secara anggaran
shade $P(|X| > a) = 0.15$ in an estimation.



- (i) $P(X < a) = P(X > 54)$
 $a = 46 - (54 - 46) = 38$
- (ii) $P(a < X < 54) = 0.86$ dan $P(X > 54) = 0.07$,
 $P(X < a) = 1 - 0.86 - 0.07 = 0.07$
- (iii) $P(a < X \leq 46) = 0.231$, maka/hence
 $P(X < a) = 0.5 - 0.231 = 0.269$
 $P(X > a) = 1 - 0.269 = 0.731$



(iii) $P(|X| > a) = P(X < -a)$ dan $P(X > a)$
 $= 0.15$



14. Suatu pemboleh ubah rawak selanjar X bertaburan normal dengan min μ dan sisihan piawai σ boleh ditukar kepada taburan normal piawai. Lengkapkan yang berikut. TP 2

SP 5.3.3

A continuous random variable X has a normal distribution with mean μ and standard deviation σ can be converted into standard normal distribution. Complete the following.

Pemboleh ubah rawak X Random variable X	Min, μ Mean, μ	Sisihan piawai, σ Standard deviation, σ	Skor-z z-score
Contoh	45	8	$z = \frac{47 - 45}{8} = 0.25$
(a) $-1.2 = \frac{X - 125}{22}$ $X = 125 + 22(-1.2)$ $= 98.6$	125	22	-1.2
(b) 6.7	$-1.9 = \frac{6.7 - \mu}{1.5}$ $\mu = 6.7 + 1.9(1.5)$ $= 9.55$	1.5	-1.9
(c) 96	88	$0.65 = \frac{96 - 88}{\sigma}$ $\sigma = \frac{8}{0.65}$ $= 12.3$	0.65

15. Cari nilai k bagi setiap yang berikut.

TP 4

SP Find the value of k for each of the following.
5.3.4

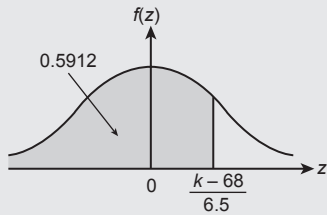
Contoh

Diberi/Given $\mu = 68, \sigma = 6.5$ dan/and
 $P(X > k) = 0.5912$.

$$P(X < k) = 0.5912$$

$$P\left(\frac{X - \mu}{\sigma} < \frac{k - 68}{6.5}\right) = 0.5912$$

$$P\left(Z < \frac{k - 68}{6.5}\right) = 0.5912$$



$$P\left(Z > \frac{k - 68}{6.5}\right) = 1 - 0.5912$$

$$= 0.4088$$

$$\frac{k - 68}{6.5} = 0.231$$

$$k = 68 + 6.5(0.231)$$

$$= 69.5$$

PENYELESAIAN

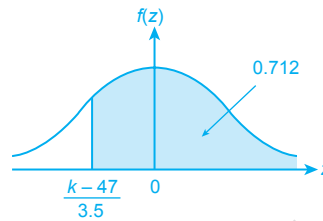


(a) Diberi/Given $\mu = 47, \sigma = 3.5$ dan/and
 $P(X > k) = 0.712$.

$$P(X > k) = 0.712$$

$$P\left(\frac{X - \mu}{\sigma} > \frac{k - 47}{3.5}\right) = 0.712$$

$$P\left(Z > \frac{k - 47}{3.5}\right) = 0.712$$



$$P\left(Z < \frac{k - 47}{3.5}\right) = 1 - 0.712$$

$$= 0.289$$

$$\frac{k - 47}{3.5} = -0.557$$

$$k = 47 - 3.5(0.557)$$

$$= 45.051$$

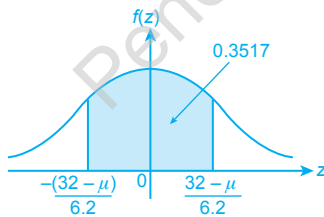
Tip Penting

Diberi luas lebih daripada 0.5, maka nilai k mesti kurang daripada min dan skor-z ialah negatif.
Given the area is more than 0.5, hence the value of k must be less than the mean and z-score is negative.

(b) Diberi/Given $\mu = k, \sigma = 6.2$ dan/and
 $P(|X| < 32) = 0.3517$.

$$P(26 < X < 32) = 0.3517$$

$$P\left(\frac{-(32 - \mu)}{\delta} < \frac{X - \mu}{\delta} < \frac{32 - \mu}{6.2}\right) = 0.3517$$



$$P\left(Z > \frac{32 - \mu}{6.2}\right) = \frac{1 - 0.3517}{2} = 0.3242$$

$$\frac{32 - k}{6.2} = 0.456$$

$$k = 32 - 6.2(0.456)$$

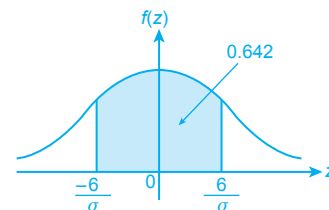
$$k = 29.17$$

(c) Diberi/Given $\mu = 54, \sigma = k$ dan/and
 $P(48 < X < 60) = 0.642$.

$$P(48 < X < 60) = 0.642$$

$$P\left(\frac{48 - 54}{\sigma} < \frac{X - \mu}{\sigma} < \frac{60 - 54}{\sigma}\right) = 0.642$$

$$P\left(\frac{-6}{\sigma} < Z < \frac{6}{\sigma}\right) = 0.642$$



$$P\left(Z > \frac{6}{\sigma}\right) = \frac{1 - 0.642}{2} = 0.179$$

$$\frac{6}{\sigma} = 0.919$$

$$\sigma = k = 6.53$$

16. Selesaikan yang berikut.

SP Solve the following.

5.2.5

Contoh

Berat pelajar di Tingkatan 5 bertaburan normal dengan min 62 kg dan sisihan piawai 10.5 kg.

The masses of students in a Form 5 class is normally distributed with mean 62 kg dan a standard deviation of 10.5 kg.

- (i) Hitung kebarangkalian satu pelajar dipilih secara rawak akan mempunyai berat antara 58 kg dan 66 kg.
Calculate the probability of a student chosen at random will have a mass between 58 kg and 66 kg.
- (ii) 70% pelajar mempunyai berat kurang daripada m . Cari nilai m .
70% of the students have masses less than m . Find the value of m .
- (iii) Cari julat antara kuartil bagi taburan berat ini.
Find the range between the quartiles for the distribution of the masses.

(i) Diberi/Given $X \sim N(62, 10.5^2)$

$$P(58 < X < 66)$$

$$= P\left(\frac{58 - 62}{10.5} < Z < \frac{66 - 62}{10.5}\right)$$

$$= P(-0.381 < Z < 0.381)$$

$$= 1 - 2P(Z > 0.381)$$

$$= 0.2968$$

Tip Penting

Lakarkan graf untuk menentukan rantau yang betul.
Sketch the graph to determine the correct region first.

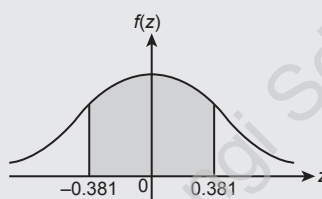
(ii) $P(X < m) = 0.7$

$$P\left(Z < \frac{m - 62}{10.5}\right) = 0.7$$

$$P\left(Z > \frac{m - 62}{10.5}\right) = 0.3$$

$$\frac{m - 62}{\sigma} = 0.524$$

$$m = 62 + 10.5(0.524) = 67.5$$

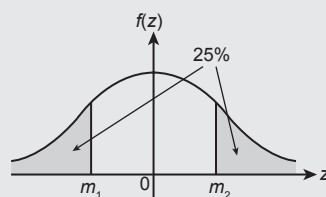


KALKULATOR



(iii) Kuartil bawah ialah 25% dan kuartil atas ialah 75%, perlu cari m_1 dan m_2

The lower quartile is 25% and the upper quartile is 75%, need to find m_1 and m_2 .



$$P(X > m_2) = P(X < m_1) = 0.25$$

$$P\left(\frac{X - \mu}{\sigma} > \frac{m_2 - 62}{10.5}\right) = 0.25$$

$$P\left(Z > \frac{m_2 - 62}{10.5}\right) = 0.25$$

$$\frac{m_2 - 62}{10.5} = 0.674$$

$$m_2 = 69.1$$

Oleh sebab graf simetri pada min, maka

Since the graph is symmetry about the mean, thus

$$m_1 = 62 - 7.1 = 54.9$$

$$\text{Julat / Range} = 69.1 - 54.9$$

$$= 14.2$$

TUTORIAL



Penyelesaian masalah
Problem solving

(a) Skor ujian kemasukan suatu kolej bertaburan normal dengan min 48 dan sisihan piawai 11. 15% calon mendapat markah kecemerlangan
The scores of a college entrance test are normally distributed with mean 48 and standard deviation of 11. 15% of the candidates obtained distinction.

(i) Cari markah minimum supaya mendapat kecemerlangan.
Find the minimum mark to get a distinction.

(ii) Kolej tidak dapat menerima calon yang mendapat markah kurang daripada 40. Jika 500 pelajar mengambil ujian itu, kira bilangan pelajar yang tidak layak masuk kolej?

The college does not accept candidates whose marks are below 40. If 500 students took the test, what is the number of students not accepted by the college?

(i) Diberi/Given $X \sim N(48, 11^2)$

$$P(X > m) = 0.15$$

$$P\left(Z > \frac{m - 48}{11}\right) = 0.15$$

$$\frac{m - 48}{11} = 1.036$$

$$m = 48 + 11(1.036) = 59.4$$

(ii) $P(X < 40) = P\left(Z < \frac{40 - 48}{11}\right) = 0.2335$

Bilangan pelajar tidak layak

Number of students not accepted into the college

$$= 0.2335 \times 500$$

$$= 117$$

(b) Gaji pekerja di sebuah kilang bertaburan normal dengan min μ dan sisihan piawai σ . Kebarangkalian gaji seorang pekerja dipilih secara rawak mendapat kurang daripada RM1 500 ialah 0.8 dan kebarangkalian mendapat gaji lebih daripada RM1 400 ialah 0.75. Hitung nilai min dan varians bagi taburan ini.

The salaries of workers in a factory are normally distributed with mean μ and standard deviation σ . The probability of a worker chosen at random whose salary is less than RM1 500 is 0.8 and the probability of a worker chosen at random whose salary is more than RM1 400 is 0.75. Calculate the mean and the standard deviation of the distribution.

Diberi/Given $X \sim N(\mu, \sigma^2)$

$$P(X < 1\,500) = 0.8$$

$$P\left(X > \frac{1500 - \mu}{\sigma}\right) = 0.8$$

$$\frac{1500 - \mu}{\sigma} = 0.842$$

$$1\,500 - \mu = 0.842\sigma \quad \dots\dots \textcircled{1}$$

$$P(X > 1\,400) = 0.75$$

$$P\left(Z > \frac{1400 - \mu}{\sigma}\right) = 0.75$$

$$\frac{1400 - \mu}{\sigma} = -0.674$$

$$1\,400 - \mu = -0.674\sigma \quad \dots\dots \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}$$

$$100 = 1.516\sigma$$

$$\sigma = 65.96$$

$$\mu = 1\,400 + 65.96(0.674)$$

$$= 1444.46$$

VIDEO



Menggunakan kalkulator saintifik untuk mencari skor-z, taburan normal.

Using scientific calculator to find z-score, normal distribution.

KBAT EKSTRA



MASTERI KE ARAH SPM

Tinggi sekumpulan penari perempuan bertaburan normal dengan min 160 cm. Diberi bahawa kebarangkalian seorang penari dipilih secara rawak mempunyai ketinggian kurang daripada 159 cm ialah 0.3542.

The heights of a group of women dancers are normally distributed with a mean of 160 cm. Given that a dancer chosen randomly has a height of less than 159 cm is 0.3542.

- (a) Jika 8 orang penari dipilih secara rawak, cari kebarangkalian bahawa
If 8 dancers are chosen at random, find the probability that
- tepat 4 orang penari mempunyai ketinggian kurang daripada 159 cm.
exactly 4 dancers are less than 159 cm tall.
 - sekurang-kurangnya 6 orang penari mempunyai tinggi kurang daripada 159 cm.
at least 6 dancers are less than 159 cm tall.
- (b) Jika seorang penari dipilih secara rawak, cari kebarangkalian bahawa tingginya kurang daripada 164 cm.
If a dancer is chosen at random, find the probability that her height is less than 164 cm.

Faham

- (a) (i) $n = 8, p = 0.3542, q = 1 - 0.3542$
 X ialah pemboleh ubah bagi penari yang mempunyai ketinggian kurang daripada 159 cm. Cari $P(X = 4)$
- (ii) $P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8)$

- (b) $X \sim N(160, \sigma^2)$
 Diberi/Given $P(X < 159) = 0.3542$
 Cari/Find $P(X < 164)$

Tulis

- (a) (i) $P(X = 4) = {}^8C_4(0.3542)^4(0.6458)^4$
 $= 0.1916$
- (ii) $P(X \geq 6)$
 $= P(X = 6) + P(X = 7) + P(X = 8)$
 $= {}^8C_6(0.3542)^6(0.6458)^2 + {}^8C_7(0.3542)^7(0.6458)^1 +$
 ${}^8C_8(0.3542)^8(0.6458)^0$
 $= 0.0269$

- (b) $X \sim N(160, \sigma^2)$
 Diberi/Given $P(X < 159) = 0.3542$
 $P\left(X < \frac{159 - 160}{\sigma}\right) = 0.3542$
 $-\frac{1}{\sigma} = -0.374$
 $\sigma = 2.674$
 $P(X < 164) = P\left(Z < \frac{164 - 160}{2.674}\right) = P\left(Z < \frac{4}{2.674}\right)$
 $= 0.9327$

CUBA DIRI

Jisim nanas dari sebuah kebun bertaburan normal dengan min 1.2 kg. Diberi bahawa kebarangkalian sebiji nanas dipilih secara rawak mempunyai jisim lebih daripada 1.32 kg ialah 0.2587.

The masses of pineapples from an orchard are normally distributed with a mean of 1.2 kg. Given that the probability of a pineapple that is chosen at random has a mass of more than 1.32 kg is 0.2587.

- (a) Jika 9 biji nanas dipilih secara rawak, cari kebarangkalian bahawa
If 9 pineapples are chosen at random, find the probability that
- tepat 5 biji nanas mempunyai jisim lebih daripada 1.32 kg.
exactly 5 pineapples have masses more than 1.32 kg.
 - selebih-lebihnya 2 biji nanas mempunyai jisim lebih daripada 1.32 kg.
at most 2 pineapples have masses more than 1.32 kg.
- (b) Jika sebiji nanas dipilih secara rawak, cari kebarangkalian bahawa jisimnya kurang daripada 1.1 kg.
If a pineapple is chosen at random, find the probability that its mass is less than 1.1 kg.

(b) 0.2949
 (a) (i) 0.0441 (ii) 0.5763
 Jawapan/Answer :

MODUL SPM

UJIAN

SKOP

HALAMAN

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Kukuh Kemahiran

• Bab 1 – Bab 8
<https://qr.pelangibooks.com/?u=TargetMatTamTg5KK>



Kertas Model SPM

<https://qr.pelangibooks.com/?u=TargetMatTamTg5KMSPM>

*Jangan lupa Pelangi Online Test (POT)
untuk latihan ekstra!*

▶▶▶ **POT**

<https://qr.pelangibooks.com/?u=POTMT5>

Enrolment key: h2&P#9b#



KERTAS 1

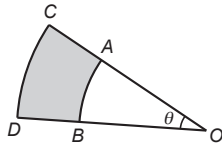
Bahagian A

1. Rajah di bawah menunjukkan sektor AOB dan COD berpusat O dan bersudut θ rad.

SP

1.3.3

The diagram below shows the sector AOB and COD with centre O and with angle θ rad.



Diberi bahawa $OA : AC = 3 : 2$, cari nisbah luas sektor AOB kepada rantau berlorek $ABCD$.

Given that $OA : AC = 3 : 2$, find the ratio of the areas of sector AOB to the shaded region of $ABCD$.

[3 markah / 3 marks]

Jawapan / Answer:

$$\text{Luas } AOB = \frac{1}{2}(3j)^2\theta$$

Area of AOB

$$\text{Luas } OCD = \frac{1}{2}(5j)^2\theta$$

Area of OCD

$$\begin{aligned} \therefore \text{luas } ABCD &= \frac{1}{2}(25j^2\theta) - \frac{1}{2}(9j^2\theta) \\ &= \frac{16}{2}j^2\theta \end{aligned}$$

Nisbah luas $AOB : ABCD$

The ratio of the area of $AOB : ABCD$

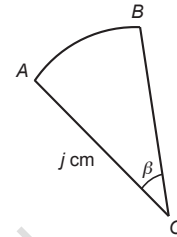
$$\begin{aligned} &= \frac{1}{2}(9j^2\theta) : \frac{1}{2}(16j^2\theta) \\ &= 9 : 16 \end{aligned}$$

2. Rajah di bawah menunjukkan sebuah sektor AOB berpusat O dan berjajari j cm dan $\angle AOB = \beta$.

SP

1.2.1

The diagram below shows a sector AOB with centre O and radius j cm and $\angle AOB = \beta$.



Diberi bahawa perimeter bagi sektor AOB ialah $(10 + \pi)$ cm dan luas sektor AOB ialah $\frac{5\pi}{2}$ cm². Tentukan nilai j dan β yang mungkin

Given that the perimeter of sector AOB is $(10 + \pi)$ cm and the area of sector AOB is $\frac{5\pi}{2}$ cm². Determine the possible values of j and β .

[4 markah / 4 marks]

Jawapan / Answer:

$$10 + \pi = 2j + j\beta$$

$$\frac{1}{2}j^2\beta = \frac{5\pi}{2}$$

$$\beta = \frac{5\pi}{j^2}$$

$$\therefore 10 + \pi = 2j + j\left(\frac{5\pi}{j^2}\right)$$

$$10j + \pi j = 2j^2 + 5\pi$$

$$2j^2 - (10 + \pi)j + 5\pi = 0$$

$$(2j - \pi)(j - 5) = 0$$

$$\therefore j = \frac{\pi}{2} \text{ atau / or } j = 5$$

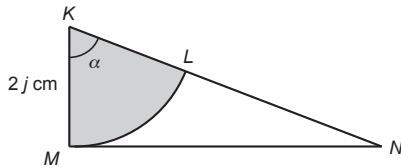
$$\begin{aligned} \beta &= \frac{5\pi}{\frac{\pi^2}{4}} = \frac{20}{\pi} \text{ atau / or } \frac{\pi}{5} \\ &= \frac{20}{\pi} \text{ rad.} \end{aligned}$$

$$\therefore j = \frac{11}{7} \text{ cm atau } 5 \text{ cm.}$$

$$\beta = \frac{20}{\pi} \text{ rad atau / or } \frac{\pi}{5} \text{ rad.}$$

- 3.** Rajah di bawah menunjukkan sebuah sektor KLM berpusat K , dengan keadaan $\angle LKM$ ialah α dan jejari $2j$ cm. KMN pula ialah sebuah segi tiga bersudut tegak, dengan keadaan L ialah titik tengah bagi KN .

The diagram below shows a sector KLM with centre K , such that $\angle LKM$ is α and the radius $2j$ cm. KMN is a right-angled triangle, such that L is the midpoint of KN .



Diberi bahawa luas sektor dan perimeter bagi KLM masing-masing ialah 21.6 cm^2 dan 19.2 cm .

Given that the area of the sector and the perimeter of KLM are 21.6 cm^2 and 19.2 cm respectively.

- (a) Bentukkan dua persamaan yang melibatkan j dan α .
Form two equations that involve j and α .
[2 markah / 2 marks]
- (b) Seterusnya, cari nilai j dan nilai α .
Then, find the value of j and of α .
[2 markah / 2 marks]
- (c) Cari panjang MN , dalam cm
Find the length of MN , in cm.
[1 markah / 1 mark]

Jawapan / Answer:

(a) $\frac{1}{2}(2j)^2\alpha = 21.6 \dots\dots \textcircled{1}$
 $4j + 2j\alpha = 19.2 \dots\dots \textcircled{2}$

(b) Daripada/From $\textcircled{1}$,
$$j\alpha = \frac{10.8}{j}$$

$$4j + 2\left(\frac{10.8}{j}\right) = 19.2$$

$$4j^2 + 21.6 - 19.2j = 0$$

$$j^2 - 4.8j + 5.4 = 0$$

$$10j^2 - 48j + 54 = 0$$

$$5j^2 - 24j + 27 = 0$$

$$(5j - 9)(j - 3) = 0$$

$$j = \frac{9}{5} \text{ atau/or } j = 3$$

$$\alpha = \frac{10.8}{\left(\frac{9}{5}\right)^2} \text{ atau/or } \alpha = \frac{10.8}{9}$$

$$\alpha = \frac{10}{3} \text{ rad atau/or } \alpha = 1.2 \text{ rad}$$

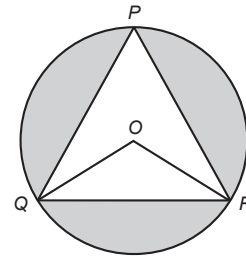
(c)
$$MN = \sqrt{12^2 - 6^2}$$

$$= \sqrt{108}$$

$$= 6\sqrt{3} \text{ cm}$$

- 4.** Rajah di bawah menunjukkan draf logo Syarikat Teknologi berbentuk bulatan berpusat O .

The diagram below shows a draft logo of a Technology company which is a circle with centre O .

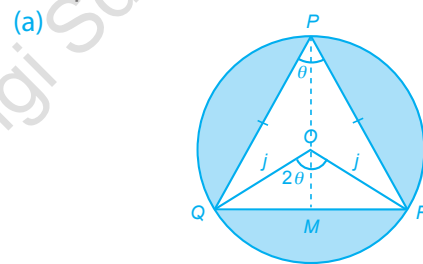


Diberi bahawa $PQ = PR$, $\angle QPR = \theta$ rad dan jejari bulatan ialah j cm.

It is given that $PQ = PR$, $\angle QPR = \theta$ rad and the radius of the circle is j cm.

- (a) Tunjukkan bahawa luas sektor $QOR = j^2\theta$
Show that the area of the sector $QOR = j^2\theta$
[2 markah / 2 marks]
- (b) Seterusnya, ungkapkan luas rantau berlorek dalam sebutan π dan j .
Then, express the area of the shaded region in terms of π and j .
[4 markah / 4 marks]

Jawapan / Answer:



$\angle QOR = 2\theta$

Daripada/From $\frac{2\theta}{2\pi} = \frac{\text{Luas/Area } QOR}{\pi j^2}$

$\therefore \text{Luas/Area } QOR = \theta j^2$

(b) $\sin \theta = \frac{QM}{OQ} \Rightarrow QM = j \sin \theta$

$\sin \frac{\theta}{2} = \frac{QM}{QP}$

$$QP = \frac{j \sin \theta}{\sin \frac{\theta}{2}} = \frac{j \left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)}{\sin \frac{\theta}{2}}$$

$$= 2j \cos \frac{\theta}{2}$$

Luas rantau berlorek/Area of the shaded region

$$= \pi j^2 - \frac{1}{2} \left(2j \cos \frac{\theta}{2} \right)^2 \sin \theta$$

$$= \pi j^2 - 2j^2 \cos^2 \frac{\theta}{2} \sin \theta$$

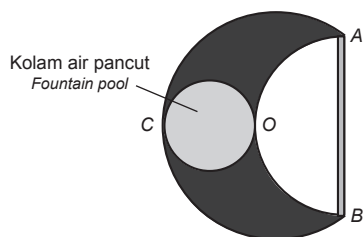
$$= j^2 \left[\pi - 2 \cos^2 \frac{\theta}{2} \sin \theta \right]$$

Bahagian B

5. Rajah di bawah menunjukkan pelan kawasan rekreasi berbentuk tembereng major sebuah bulatan. **KBAT** Menganalisis

SP
1.4.1

The diagram below shows a plan for a recreation region in the shape of a major segment of a circle.



Pintu masuknya ialah AB, dengan lebar 20 m. AB juga ialah diameter semibulatan, dengan keadaan lilitannya melalui pusat O tembereng itu. Kolam air pancut ialah kawasan yang berlorek.

The entrance is AB, with a width of 20 m. AB is also the diameter of a semicircle, such that the circumference passes through the centre O of the segment. The fountain pool is the shaded region.

(a) Cari jejari tembereng major kawasan rekreasi.
Find the radius of the major segment of the recreation region.

[1 markah / 1 mark]

(b) Cari perimeter keseluruhan kawasan rekreasi itu dalam sebutan π .
Find the perimeter of the entire recreation region in terms of π .

[2 markah / 2 marks]

(c) Bahagian yang berwarna hitam ditanam dengan rumput. Cari luas yang diliputi oleh rumput dalam sebutan π .
The black part is planted with grass. Find the area covered by the grass in terms of π .

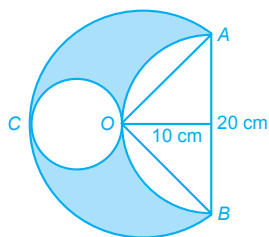
[4 markah / 4 marks]

(d) Seterusnya, cari kos untuk menanam rumput jika harganya ialah RM60 per m^2 .
Then, find the cost to plant the grass if the price is RM60 per m^2 .

[1 markah / 1 mark]

Jawapan / Answer:

(a)



(a) $OA = \sqrt{10^2 + 10^2}$
 $= 10\sqrt{2} \text{ m}$

(b) Perimeter

$$= \frac{3}{4}[2\pi(10\sqrt{2})] + 20$$

$$= [15\sqrt{2}\pi + 20] \text{ m}$$

(c) Luas diliputi oleh rumput

Area covered by grass

$$= \frac{3}{4}\pi(10\sqrt{2})^2 + \frac{1}{2}(10\sqrt{2})^2 - \frac{1}{2}\pi(10)^2 - \pi(5\sqrt{2})^2$$

$$= \frac{3}{4}\pi(200) + 100 - 50\pi - 50\pi$$

$$= 50\pi + 100$$

$$= 50(\pi + 2) \text{ m}^2$$

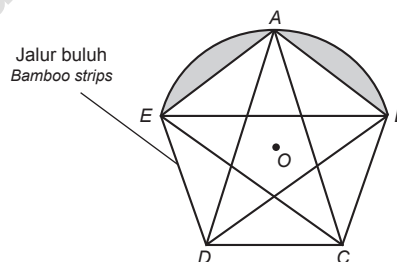
(d) Kos/Cost = $50(\pi + 2)(\text{RM}60)$

$$= \text{RM}15\,426$$

6. Rajah di bawah menunjukkan sebuah layang-layang berbentuk pentagon sekata ABCDE yang diterapkan di dalam satu bulatan berjejari 30 cm dari pusat O. **KBAT** Menganalisis

SP
1.3.2

The diagram below shows a kite in the shape of a regular pentagon ABCDE that is inscribed in a circle with a radius of 30 cm from the center O.



Kertas yang berwarna digunakan untuk membina kedua-dua tembereng yang berlorek berpusat O juga. Jalur buluh digunakan untuk membina bingkai dan bentuknya.

The paper with colour is used to construct the two shaded sections centered at O as well. Bamboo strips are used to build the frame and shape.

(a) Cari sudut $\angle AOB$, dalam sebutan π .

Find the angle $\angle AOB$, in terms of π .

[2 markah / 2 marks]

(b) Tentukan sama ada 5 m jalur buluh mencukupi untuk membuat bingkai layang-layang itu.

Determine whether the 5 m bamboo strips are enough to make the frame of the kite.

[4 markah / 4 marks]

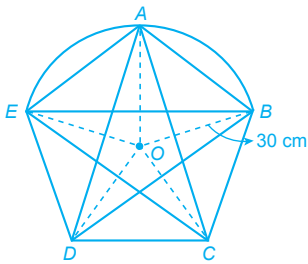
(c) Kertas kuning akan digunakan untuk membentuk tembereng AE dan AB. Cari jumlah luas tembereng itu.

Yellow paper is used to cover the segments AE and AB. Find the total area of the segment.

[2 markah / 2 marks]

Jawapan / Answer:

(a)



$$\begin{aligned} \text{(a) } \angle AOB &= \frac{360^\circ}{5} = 72^\circ \\ &= \frac{2\pi}{5} \end{aligned}$$

$$\begin{aligned} \text{(b) panjang lengkok AB} \\ \text{arc length of AB} \\ &= 30 \left(\frac{2\pi}{5} \right) = 12\pi \end{aligned}$$

$$\begin{aligned} \text{Panjang AB} &= \sqrt{30^2 + 30^2 - 2(30)^2 \cos 72^\circ} \\ \text{Length of AB} &= 35.27 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Panjang AC} &= \sqrt{30^2 + 30^2 - 2(30)^2 \cos 144^\circ} \\ \text{Length of AC} &= 57.06 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Jumlah panjang jalur buluh} \\ \text{Total length of bamboo strips} \\ &= 2(12\pi) + 5(35.27) + 5(57.06) \\ &= 537.05 \text{ cm} \\ &= 5.37 \text{ m} > 5 \text{ m (Tidak mencukupi / Not enough)} \end{aligned}$$

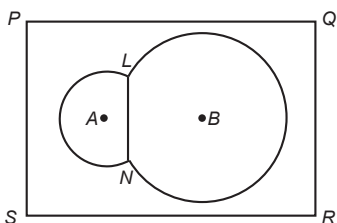
$$\begin{aligned} \text{(c) Luas tembereng AB dan AE} \\ \text{Area of the segments AB and AE} \\ &= 2 \left[\frac{1}{2} (30)^2 \left(\frac{2\pi}{5} \right) - \frac{1}{2} (30)^2 \sin 72^\circ \right] \\ &= 275.17 \text{ cm}^2 \end{aligned}$$

KERTAS 2

Bahagian A

1. Fanny mempunyai sekeping tanah berbentuk segi empat tepat PQRS, dengan luas $H \text{ m}^2$, seperti yang ditunjukkan dalam Rajah. Dia bercadang untuk menanam bunga ros dalam tembereng kecil berpusat A, manakala dalam tembereng besar berpusat B, dia akan menanam bunga orkid.

Fanny has a piece of land in the shape of rectangle PQRS, with an area of $H \text{ m}^2$, as shown in the diagram. She plans to plant roses in the small segment centred at A, while in the large segment centred at B, she will plant orchids.



Nisbah jejari sektor kecil kepada sektor besar ialah $1 : \sqrt{3}$. Diberi bahawa $\angle_{\text{minor}} LAN = \frac{2\pi}{3}$ dan $\angle_{\text{major}} LBN = \frac{5\pi}{3}$.

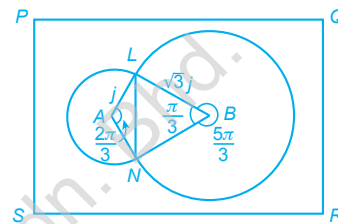
Jika $j \text{ m}$ ialah jejari bagi tembereng berpusat A, tunjukkan bahawa luas kawasan yang tidak ditanam dengan ros atau orkid ialah $\left[H - j^2 \left(\frac{19\pi}{6} + \sqrt{3} \right) \right] \text{ m}^2$.

The ratio of the radius of the small sector to the large sector is $1 : \sqrt{3}$. It is given that $\angle_{\text{minor}} LAN = \frac{2\pi}{3}$ and $\angle_{\text{major}} LBN = \frac{5\pi}{3}$.

If $j \text{ m}$ is the radius of the segment with centre A, show that the area of the region that is not planted with roses or orchids is $\left[H - j^2 \left(\frac{19\pi}{6} + \sqrt{3} \right) \right] \text{ m}^2$.

[6 markah / 6 marks]

Jawapan / Answer:



Luas kawasan ditanam dengan ros
Area of the region planted with roses

$$\begin{aligned} &= \frac{1}{2} j^2 \left(\frac{4\pi}{3} \right) + \frac{1}{2} j^2 \sin 120^\circ \\ &= \frac{2\pi}{3} j^2 + \frac{1}{4} j^2 \sqrt{3} \end{aligned}$$

Luas kawasan ditanam dengan orkid
Area of the region planted with orchids

$$\begin{aligned} &= \frac{1}{2} j^2 \left(\frac{5\pi}{3} \right) (3) + \frac{1}{2} (\sqrt{3}j)^2 \sin 60^\circ \\ &= \frac{5\pi}{2} j^2 + \frac{3j^2}{2} \left(\frac{\sqrt{3}}{2} \right) \\ &= \frac{5\pi}{2} j^2 + \frac{3\sqrt{3}}{4} j^2 \end{aligned}$$

Jumlah luas kawasan ditanam dengan ros dan orkid

Total area of the region planted with roses and orchids

$$\begin{aligned} &= \frac{2\pi}{3} j^2 + \frac{5\pi}{2} j^2 + \frac{\sqrt{3}}{4} j^2 + \frac{3\sqrt{3}}{4} j^2 \\ &= \frac{19\pi}{6} j^2 + \sqrt{3} j^2 \end{aligned}$$

∴ Luas kawasan tidak ditanam dengan ros dan orkid
Area of the region that is not planted with roses and orchids

$$= \left[H - j^2 \left(\frac{19\pi}{6} + \sqrt{3} \right) \right] \text{ m}^2$$

UJIAN 2

KERTAS 1

Bahagian A

1. Cari $\lim_{x \rightarrow 0} \frac{x}{3 - \sqrt{x+9}}$.

SP

2.1.2 Find $\lim_{x \rightarrow 0} \frac{x}{3 - \sqrt{x+9}}$.

[3 markah / 3 marks]

Jawapan / Answer:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{x}{3 - \sqrt{x+9}} \\ &= \lim_{x \rightarrow 0} \frac{x(3 + \sqrt{x+9})}{(3 - \sqrt{x+9})(3 + \sqrt{x+9})} \\ &= \lim_{x \rightarrow 0} \frac{x(3 + \sqrt{x+9})}{9 - (x+9)} \\ &= \lim_{x \rightarrow 0} \frac{x(3 + \sqrt{x+9})}{(-x)} \\ &= -6 \end{aligned}$$

2. Luas permukaan sebuah sfera menyusut dengan kadar $0.9\pi \text{ cm}^2 \text{ s}^{-1}$ apabila jejaringnya ialah 6 cm. Cari $\frac{dA}{dt}$ when its radius is 6 cm. Find

SP
2.4.6
2.4.7

(a) kadar perubahan isi padu sfera tersebut pada ketika itu.
the rate of change of the volume of the sphere at that time.

[4 markah / 4 marks]

(b) peratus perubahan luas permukaan pada ketika itu jika peratus perubahan bagi jejaringnya ialah 0.5%.
the percentage change of the surface area at that time if the percentage change of the radius is 0.5%.

[3 markah / 3 marks]

Jawapan / Answer:

(a) Luas / Area $A = 4\pi r^2$

$$\frac{dA}{dr} = 8\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$0.9\pi = 8\pi(6) \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{0.9}{48} = \frac{3}{160} \text{ cm s}^{-1}$$

Isi padu sfera $V = \frac{4}{3}\pi r^3$
Volume of the sphere

$$\frac{dV}{dr} = 4\pi r^2$$

$$\begin{aligned} \frac{dV}{dt} &= 4\pi r^2 \times \frac{dr}{dt} \\ &= 4\pi(6)^2 \left(\frac{3}{160}\right) \\ &= 2.7\pi \text{ cm}^3 \text{ s}^{-1} \end{aligned}$$

(b) Diberi $\frac{dr}{r} \times 100\% = 0.5\%$

$$\begin{aligned} \frac{dA}{A} \times 100\% &= \frac{8\pi r \times dr}{4\pi r^2} \times 100\% \\ &= 2 \left(\frac{dr}{r} \times 100\%\right) \\ &= 2(0.5\%) \\ &= 1\% \end{aligned}$$

3. (a) Cari titik pusingan bagi lengkung $y = \frac{2+x^3}{x}$. Seterusnya, tentukan sama ada titik pusingan ialah maksimum atau minimum.

SP
2.4.4
2.4.5

Find the turning point of the curve $y = \frac{2+x^3}{x}$. Then, determine whether the turning point is a maximum or a minimum.

[3 markah / 3 marks]

(b) Tunjukkan bahawa kecerunan lengkung adalah negatif bagi semua nilai x.
Show that the gradient of the curve is negative for all the values of x.

[3 markah / 3 marks]

Jawapan / Answer:

(a) $y = \frac{2+x^3}{x} = 2x^{-1} + x^2$

$$\frac{dy}{dx} = -2x^{-2} + 2x$$

$$= -\frac{2}{x^2} + 2x$$

Apabila/When $\frac{dy}{dx} = 0$

$$2x = \frac{2}{x^2}$$

$$x^3 = 1$$

$$x = 1$$

$$y = \frac{2+1^3}{1} = 3$$

$$\frac{d^2y}{dx^2} = 4x^{-3} + 2$$

Apabila/When $x = 1$, $\frac{d^2y}{dx^2} = 6$

$$\frac{d^2y}{dx^2} > 0$$

Titik pusingan (1, 3) adalah maksimum
Turning point (1, 3) is maximum.

$$(b) \frac{dy}{dx} = -\frac{2}{x^2} + 2x$$

Apabila/When $x < 0$, $-\frac{2}{x^2} < 0$ dan/and $2x < 0$

Maka/Thus, $x < 0$, $-\frac{2}{x^2} + 2x < 0$

$\frac{dy}{dx}$ adalah sentiasa negatif jika $x < 0$.

$\frac{dy}{dx}$ is always negative if $x < 0$.

Bahagian B

4. (a) Cari persamaan tangen bagi lengkung $y = (x - 5)(x + 1)$ yang selari dengan paksi-x.
SP Find the equation of the tangent of the curve $y = (x - 5)(x + 1)$ that is parallel to the x-axis.
 2.4.2
 2.4.3 [4 markah / 4 marks]

- (b) Cari nilai k supaya garis $y = -\frac{1}{2}x + k$ adalah normal kepada lengkung $y = 2x^2 - 3$.
 Find the value of k so that the line $y = -\frac{1}{2}x + k$ is normal to the curve $y = 2x^2 - 3$.
 [4 markah / 4 marks]

Jawapan / Answer:

(a) $y = (x - 5)(x + 1)$
 $= x^2 - 4x - 5$

$$\frac{dy}{dx} = 2x - 4$$

Jika selari dengan paksi-x, $\frac{dy}{dx} = 0$.
 If parallel to the x-axis

$$\therefore 2x - 4 = 0$$

$$x = 2$$

$$y = (2 - 5)(2 + 1) = -9$$

\therefore Persamaan tangen ialah $y = -9$
 The equation of the tangent is $y = -9$

(b) $y = 2x^2 - 3$

$$\frac{dy}{dx} = 4x$$

Kecerunan normal = $-\frac{1}{2}$
 The gradient of the normal

$$\therefore \frac{dy}{dx} = 4x = 2$$

$$x = \frac{1}{2}$$

$$\therefore y = 2\left(\frac{1}{2}\right)^2 - 3$$

$$= -\frac{5}{2}$$

\therefore Persamaan normal
 The equation of the normal

$$y = -\frac{1}{2}x + k$$

$$-\frac{5}{2} = -\frac{1}{2}\left(\frac{1}{2}\right) + k$$

$$-\frac{5}{2} + \frac{1}{4} = k$$

$$-\frac{9}{4} = k$$

KERTAS 2

Bahagian A

1. Fungsi terbitan bagi $y = ax^2 + bx + c$ ialah $4x + 2$.
SP Diberi bahawa y mempunyai nilai minimum 1.
 2.4.1 The derivative function of $y = ax^2 + bx + c$ is $4x + 2$. Given that y
 2.4.8 has a minimum value of 1.

- (a) Cari nilai-nilai a , b dan c .
 Find the values of a , b , and c .

[4 markah / 4 marks]

- (b) Seterusnya, tentukan perubahan hampir bagi y dalam sebutan α apabila x berubah daripada α kepada $\alpha + \delta\alpha$.
 Then, determine the approximate change of y in terms of α when x changes from α to $\alpha + \delta\alpha$.

[2 markah / 2 marks]

Jawapan / Answer:

(a) $y = ax^2 + bx + c$

$$\frac{dy}{dx} = 2ax + b = 4x + 2$$

$$\therefore a = 2, \text{ dan } b = 2$$

$$4x + 2 = 0$$

$$x = -\frac{1}{2}$$

Apabila/When $x = -\frac{1}{2}$, $y = 1$

$$\therefore 1 = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + c$$

$$1 = \frac{1}{2} - 1 + c$$

$$2 - \frac{1}{2} = c$$

$$c = \frac{3}{2}$$

(b) $\frac{dy}{dx} = 4x + 2$

$$\delta y = \frac{dy}{dx} \times \delta x$$

$$\therefore \delta y = (4x + 2) \delta x$$

Apabila/When $x = \alpha$

$$\delta y = (4\alpha + 2) \delta\alpha$$

UJIAN 3

KERTAS 1

Bahagian A

1. Diberi bahawa $\frac{d}{dx} \left[\frac{3x-5}{(1-4x)^2} \right] = \frac{1}{2}g(x)$, cari

SP

3.3.1 It is given that $\frac{d}{dx} \left[\frac{3x-5}{(1-4x)^2} \right] = \frac{1}{2}g(x)$, find

(a) $\int_0^2 g(x) dx$ [2 markah / 2 marks]

(b) $\int_0^1 [x - 2g(x)] dx$ [2 markah / 2 marks]

Jawapan / Answer:

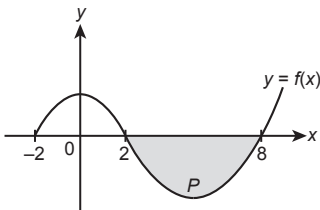
(a) $\int \frac{1}{2}g(x) dx = \frac{3x-5}{(1-4x)^2}$
 $\int_0^2 g(x) dx = \left[\frac{2(3x-5)}{(1-4x)^2} \right]_0^2$
 $= \frac{2}{49} - \frac{2(-5)}{1}$
 $= 10\frac{2}{49}$

(b) $\int_0^1 x dx - 2 \int_0^1 g(x) dx$
 $= \left[\frac{x^2}{2} \right]_0^1 - \left[\frac{4(3x-5)}{(1-4x)^2} \right]_0^1$
 $= \frac{1}{2} - \left[\frac{4(-2)}{9} - \frac{4(-5)}{1} \right]$
 $= \frac{1}{2} - \left[\frac{-8}{9} + 20 \right]$
 $= -18\frac{11}{18}$

2. Rajah di bawah menunjukkan sebahagian graf

SP $y = f(x)$.

3.3.1 The diagram below shows part of the graph $y = f(x)$.



Diberi bahawa luas P ialah 7 unit² dan $\int_{-2}^8 f(x) dx = -2$. Nyatakan nilai bagi

It is given the area of P is 7 unit² and $\int_{-2}^8 f(x) dx = -2$. State the value of

(a) $\int_2^8 f(x) dx$ [1 markah / 1 mark]

(b) $\int_{-2}^2 \frac{1}{2}f(x) dx$ [2 markah / 2 marks]

Jawapan / Answer:

(a) $\int_2^8 f(x) dx = -7$

(b) $\int_{-2}^2 f(x) dx + \int_2^8 f(x) dx = -2$
 $\int_{-2}^2 f(x) dx = -2 - (-7) = 5$
 $\frac{1}{2} \int_{-2}^2 f(x) dx = \frac{5}{2}$

3. Suatu lengkung melalui titik P(4, -3), mempunyai fungsi kecerunan $2 - 3x^2$. Cari.

SP

3.2.4 A curve passes through the point P(4, -3), has gradient function $2 - 3x^2$. Find

(a) persamaan lengkung.
the equation of the curve. [2 markah / 2 marks]

(b) persamaan normal pada titik P.
the equation of the normal at point P. [3 markah / 3 marks]

Jawapan / Answer:

(a) $\frac{dy}{dx} = 2 - 3x^2$
 $\int \frac{dy}{dx} dx = \int 2 - 3x^2 dx$
 $y = 2x - x^3 + c$
 $-3 = 2(4) - 4^3 + c$
 $c = 53$
 $y = 2x - x^3 + 53$

(b) $\frac{dy}{dx} = 2 - 3(4)^2$
 $= -46$

Kecerunan normal / Gradient of the normal = $\frac{-1}{-46}$
 $= \frac{1}{46}$

$y = \frac{1}{46}x + c$
 $-3 = \frac{1}{46}(4) + c$

$$c = -\frac{71}{23}$$

$$y = \frac{1}{46}x - \frac{71}{23}$$

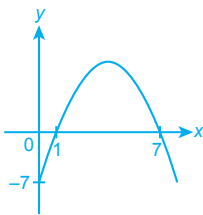
$$46y = x - 142$$

Bahagian B

4. (a) Lakar graf $y = -x^2 + 8x - 7$.
 Sketch the graph of $y = -x^2 + 8x - 7$.
 [2 markah / 2 marks]
- (b) Seterusnya, cari luas yang dibatasi oleh lengkung dan paksi-x.
 Hence, find the area bounded by the curve and the x-axis.
 [3 markah / 3 marks]
- (c) Cari isi padu janaan, dalam sebutan π , apabila rantau yang dibatasi oleh lengkung, paksi-x dan paksi-y dikisarkan melalui 360° pada paksi-x.
 Find the volume generated, in terms of π , when the region bounded by the curve, the x-axis and the y-axis is rotated through 360° about the x-axis.
 [3 markah / 3 marks]

Jawapan / Answer:

- (a) $y = -x^2 + 8x - 7$
 $y = (1 - x)(x - 7)$
 Apabila/When $y = 0$,
 $x = 1$ atau/or $x = 7$



(b) Luas rantau / Area of the region

$$= \int_1^7 (-x^2 + 8x - 7) dx$$

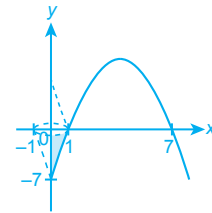
$$= \left[-\frac{x^3}{3} + 4x^2 - 7x \right]_1^7$$

$$= \left[-\frac{7^3}{3} + 4(7)^2 - 7(7) \right] - \left[-\frac{1}{3} + 4 - 7 \right]$$

$$= 32\frac{2}{3} - \left(-3\frac{1}{3}\right)$$

$$= 36 \text{ unit}^2$$

(c)



Isi padu janaan/Volume generated

$$= \pi \int_0^1 (-x^2 + 8x - 7)^2 dx$$

$$= \pi \int_0^1 (x^4 - 8x^3 + 7x^2 - 8x^3 + 64x^2 - 56x + 7x^2 - 56x + 49) dx$$

$$= \pi \int_0^1 (x^4 - 16x^3 + 78x^2 - 112x + 49) dx$$

$$= \pi \left[\frac{x^5}{5} - 4x^4 + 26x^3 - 56x^2 + 49x \right]_0^1$$

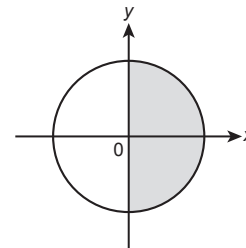
$$= \pi \left(\frac{1}{5} - 4 + 26 - 56 + 49 \right)$$

$$= 15\frac{1}{5}\pi \text{ unit}^3$$

KERTAS 2

Bahagian A

1. Rajah di bawah menunjukkan suatu bulatan mempunyai persamaan $x^2 + y^2 = r^2$, dengan keadaan r ialah jejari bulatan itu.
 The diagram below shows a circle with the equation $x^2 + y^2 = r^2$, such that r is the radius of the circle.



Buktikan bahawa isi padu sfera yang dijanakan apabila luas berlorek itu ialah
 Prove that the volume of the sphere that is generated when the shaded area is

- (a) $\frac{4}{3}\pi r^3$ unit³ apabila diputar 360° pada paksi-y.
 $\frac{4}{3}\pi r^3$ unit³ when it is revolved through 360° about the y-axis.
 [3 markah / 3 marks]
- (b) $\frac{2}{3}\pi r^3$ unit³ apabila diputar 180° pada paksi-x.
 $\frac{2}{3}\pi r^3$ unit³ when it is revolved through 180° about the x-axis.

[3 markah / 3 marks]

Jawapan / Answer:

(a) Isi padu janaan/Volume generated

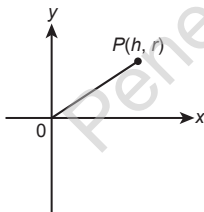
$$\begin{aligned} &= \pi \int_{-r}^r (r^2 - y^2) dy \\ &= \pi \left[r^2 y - \frac{y^3}{3} \right]_{-r}^r \\ &= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - \left(-r^3 + \frac{r^3}{3} \right) \right] \\ &= \pi \left[2r^3 - \frac{2r^3}{3} \right] \\ &= \pi \left[\frac{4r^3}{3} \right] \\ &= \frac{4}{3} \pi r^3 \text{ unit}^3 \end{aligned}$$

(b) Isi padu janaan / Volume generated

$$\begin{aligned} &= \pi \int_0^r (r^2 - x^2) dx \\ &= \pi \left[r^2 x - \frac{x^3}{3} \right]_0^r \\ &= \pi \left[r^3 - \frac{r^3}{3} \right] \\ &= \frac{2}{3} \pi r^3 \text{ unit}^3 \end{aligned}$$

2. Rajah di bawah menunjukkan garis lurus OP melalui titik $P(h, r)$.

SP 3.3.5 The diagram below shows a straight line OP passes through the point $P(h, r)$.



(a) Nyatakan persamaan garis lurus itu dalam sebutan h dan r .

State the equation of the line in terms of h and r .

[2 markah / 2 marks]

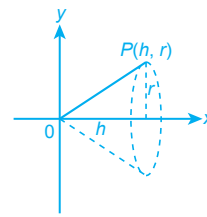
(b) Cari isi padu janaan, dalam sebutan π , h dan r , apabila rantau yang dibatasi oleh garis OP , $x = h$ dan paksi- x dikisarkan melalui 360° pada paksi- x .

Find the volume generated, in terms of π , h and r , when the region bounded by the line OP , $x = h$ and x -axis is revolved through 360° about the x -axis.

[4 markah / 4 marks]

Jawapan / Answer:

(a)



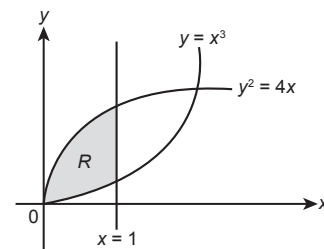
$$y = \frac{rx}{h}$$

(b) Isi padu janaan/Volume generated

$$\begin{aligned} &= \pi \int_0^h \left(\frac{r}{h} x \right)^2 dx \\ &= \frac{\pi r^2}{h^2} \int_0^h x^2 dx \\ &= \frac{\pi r^2}{h^2} \left[\frac{x^3}{3} \right]_0^h \\ &= \frac{\pi r^2}{h^2} \left[\frac{h^3}{3} \right] \\ &= \frac{1}{3} \pi r^2 h \text{ unit}^3 \end{aligned}$$

3. Rajah di bawah menunjukkan rantau berlorek R dibatasi oleh lengkung $y = x^3$, $y^2 = 4x$ dan garis lurus $x = 1$.

SP 3.3.1 3.3.3 3.3.5 The diagram below shows a shaded region R bounded by the curve $y = x^3$, $y^2 = 4x$ and the line $x = 1$.



(a) Tunjukkan bahawa luas rantau berlorek R ialah 1 unit².

Show that the area of the shaded region R is 1 unit².

[3 markah / 3 marks]

(b) Cari isi padu janaan, dalam sebutan π , apabila rantau berlorek R dikisarkan melalui empat sudut tegak pada paksi- x .

Find the volume generated, in terms of π , when the shaded region R is rotated through the four angles perpendicular to the x -axis.

[3 markah / 3 marks]

Jawapan / Answer:

(a) Luas rantau berlorek R / Area of shaded region R

$$\begin{aligned} &= \int_0^1 (\sqrt{4x} - x^3) dx \\ &= \int_0^1 (2x^{\frac{1}{2}} - x^3) dx \\ &= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{x^4}{4} \right]_0^1 \\ &= \left[\frac{4}{3} - \frac{1}{4} \right] - 0 \\ &= 1\frac{1}{12} \\ &= 1 \text{ unit}^2 \end{aligned}$$

(b) Isi padu janaan / Volume generated

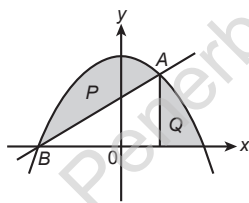
$$\begin{aligned} &= \pi \int_0^h 4x dx - \pi \int_0^h x^6 dx \\ &= \pi \left[2x^2 \right]_0^1 - \pi \left[\frac{x^7}{7} \right]_0^1 \\ &= 2\pi - \frac{\pi}{7} \\ &= \frac{13}{7}\pi \text{ unit}^3 \end{aligned}$$

Bahagian B

4. Rajah di bawah menunjukkan garis lurus $y = x + 4$ menyilang lengkung $y = -\frac{1}{2}x^2 + 8$ pada A dan B .

SP 3.4.2

The diagram below shows a line $y = x + 4$ intersects the curve $y = -\frac{1}{2}x^2 + 8$ at A and B .



- (a) Cari koordinat A dan B .
Find the coordinates of A and B .
[3 markah / 3 marks]
- (b) Hitung luas berlorek P yang dibatasi oleh lengkung dan garis lurus.
Calculate the shaded area P which is bounded by the curve and the line.
[3 markah / 3 marks]
- (c) Hitung isi padu janaan apabila luas berlorek Q yang dibatasi oleh lengkung, paksi- x dan garis mencancang yang melalui titik A diputarakan 360° pada paksi- x .
Calculate the volume generated when the shaded area Q which is bounded by the curve, the x -axis and the vertical line through A is rotated 360° about the x -axis.
[4 markah / 4 marks]

Jawapan / Answer:

(a) $\frac{1}{2}x^2 + x - 4 = 0$
 $x^2 + 2x - 8 = 0$
 $(x - 2)(x + 4) = 0$
 $\therefore x = 2 ; -4$
 $y = 6 ; 0$
 $\therefore A(2, 6) \quad B(-4, 0)$

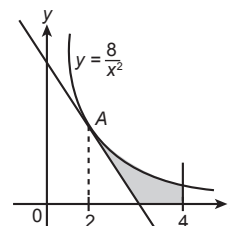
(b) Luas berlorek P = $\int_{-4}^2 \left[8 - \frac{1}{2}x^2 \right] dx - \frac{1}{2}(6)(6)$
 The shaded area P
 $= \left[8x - \frac{1}{6}x^3 \right]_{-4}^2 - 18$
 $= \left[\left(16 - \frac{4}{3} \right) - \left(-32 + \frac{32}{3} \right) \right]$
 $= 36 - 18$
 $= 18 \text{ unit}^2$

(c) Isi padu janaan = $\pi \int_2^4 y^2 dx$
 The volume generated
 $= \pi \int_2^4 \left(8 - \frac{1}{2}x^2 \right)^2 dx$
 $= \pi \int_2^4 \left(64 - 8x^2 + \frac{1}{4}x^4 \right) dx$
 $= \pi \left[64x - \frac{8}{3}x^3 + \frac{1}{20}x^5 \right]_2^4$
 $= 28\frac{4}{15} \text{ unit}^3$

5. Rajah di bawah menunjukkan lengkung $y = \frac{8}{x^2}$ dan garis lurus $y = mx + c$ adalah tangen kepada lengkung pada A dengan $x = 2$.

SP 3.4.2

The diagram below shows a curve $y = \frac{8}{x^2}$ and the line $y = mx + c$ is a tangent to the curve at A with $x = 2$.



- (a) Cari nilai m dan nilai c .
Find the value of m and of c . [3 markah / 3 marks]
- (b) Cari luas yang dibatasi oleh lengkung, paksi- x , tangen di A dan $x = 4$.
Find the area bounded by the curve, the x -axis, the tangent at A and $x = 4$.
[3 markah / 3 marks]

UJIAN 4

KERTAS 1

Bahagian A

1. Sofia telah mendaftar akaun media social yang baharu. Dia perlu menetapkan kata laluan dengan memilih 5 aksara daripada 12 pilihan aksara yang diberi dalam Jadual di bawah.

SP
4.1.2
4.1.3

Sofia has registered a new social media account. She needs to create a password using 5 characters, which are to be chosen from the characters displayed in the table below.

KBAT Mengaplikasi

Huruf besar Capital letters	Huruf kecil Small letters	Nombor Numbers	Simbol Symbols
W, R, S, P	q, k	4, 7, 9	&, *, #

Cari bilangan kata laluan berbeza dapat dibentuk jika

Find the number of ways to create the password if

- (a) tiga aksara terakhir adalah R&4 tanpa ulangan. Ulangan tidak dibenarkan.
the last three characters are R&4 without repetition. Repetition is not allowed.

[2 markah / 2 marks]

- (b) aksara pertama mesti huruf besar diikuti oleh simbol dan berakhir dengan nombor.
the first character must be a capital letter followed by a symbol and end with a number.

[2 markah / 2 marks]

Jawapan / Answer:

(a) Bilangan cara / Number of ways
 $= {}^9P_3$
 $= 72$

(b) Bilangan cara / Number of ways
 $= 4 \times 3 \times {}^9P_2 \times 3$
 $= 2\,592$

2. Satu jawatankuasa yang terdiri daripada 7 orang akan dilantik daripada 5 lelaki dan 8 perempuan.
A committee of 7 people will to be appointed from 5 men and 8 women.

SP
4.2.2

- (a) Cari bilangan cara jawatankuasa dapat dibentuk sekiranya

Find the number of ways the committee can be formed if

- (i) tiada syarat dikenakan.
no condition is imposed.
(ii) jawatankuasa mesti mengandungi selebih-lebihnya 4 orang perempuan.
the committee must consist of at most 4 women.

[4 markah / 4 marks]

- (b) Semua ahli jawatankuasa akan menghadiri suatu jamuan makan malam dan akan duduk di sebuah meja bulat. Hitung bilangan cara boleh menyusun mereka jika pengerusi, timbalan pengerusi dan setiausaha perlu duduk bersama.

All the committee members will attend a dinner and will be seated at a round table. Calculate the number of ways to arrange them if the chairman, vice chairman and the secretary must be seated together.

[3 markah / 3 marks]

Jawapan / Answer:

(a) (i) ${}^{13}C_7 = 1\,716$
 (ii) ${}^8C_4 \times {}^5C_3 + {}^8C_3 \times {}^5C_4 + {}^8C_2 \times {}^5C_5$
 $= 1\,008$

(b) Bilangan cara / Number of ways
 $= (5 - 1)! \times 3!$
 $= 144$

3. Sekumpulan murid yang terdiri daripada 4 orang lelaki dan 4 orang perempuan menghadiri sebuah majlis.

SP
4.2.3

A group of pupils consisting of 4 boys and 4 girls attended a function.

- (a) Dalam majlis tersebut, murid diminta bergambar dalam satu baris. Cari bilangan cara menyusun mereka supaya semua perempuan berdiri di bahagian tengah barisan.

In the function, the pupils are asked to have a photo taken in a row. Find the number of ways to arrange them so that all the girls stand in the middle of the row.

[2 markah / 2 marks]

- (b) Selepas tamat sesi bergambar, murid diminta duduk pada sebuah meja bulat. Cari bilangan cara menyusun mereka jika susunan mesti berselang seli.

After the photo session, the pupils were asked to sit at a round table. Find the number of ways to arrange them if the arrangement must alternate.

[3 markah / 3 marks]

- (c) Murid juga terlibat dalam satu tarian yang terdiri daripada 3 orang lelaki dan 3 orang perempuan. Cari bilangan cara memilih mereka jika seorang lelaki dan seorang perempuan tertentu mesti dipilih.

The pupils are also involved in a dance consisting of 3 boys and 3 girls. Find the number of ways to choose them if a special boy and a girl must be chosen.

[2 markah / 2 marks]

Jawapan / Answer:

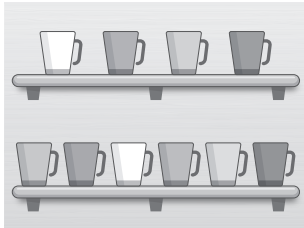
- (a) $4! \times 4! = 576$
 (b) $(4 - 1)! \times 4! = 144$
 (c) ${}^3C_2 \times {}^3C_2 = 9$

Bahagian B

4. Rajah di bawah menunjukkan 10 cawan yang berbeza warna pada dua tingkat rak di dapur Diana.

SP
4.1.3
4.2.3

The diagram below shows 10 cups of different colours on the two levels of shelves in Diana's kitchen.



- (a) Cari bilangan cara menyusun semua cawan itu jika bilangan cawan di tingkat atas adalah kurang daripada bilangan cawan di tingkat bawah pada rak dapur itu.

Find the number of ways to arrange all the cups if the number of cups on the upper shelf is less than the number of cups on the lower shelf on the kitchen shelf.

[3 markah / 3 marks]

- (b) Jika bilangan cawan di tingkat atas ialah 4 dan Diana ingin memilih 2 cawan dari tingkat atas dan 3 cawan dari tingkat bawah, cari bilangan cara Diana dapat memilih 5 cawan itu.

If the number of cups on the upper shelf is 4 and Diana wants to choose 2 cups from the upper shelf and 3 cups from the lower shelf, find the number of ways Diana can choose the 5 cups.

[2 markah / 2 marks]

- (c) Cawan yang telah dipilih dari bahagian (b) akan disusun pada sebuah meja bulat. Cari bilangan cara menyusun 5 cawan itu jika 2 cawan tertentu mesti bersebelahan.

The chosen cups in part (b) are to be arranged on a round table. Find the number of ways to arrange these 5 cups if 2 special cups must be side by side.

[3 markah / 3 marks]

Jawapan / Answer:

- (a) Bilangan cara / Number of ways
 $= {}^{10}C_4 \times 4! \times 6! + {}^{10}C_3 \times 3! \times 7! + {}^{10}C_2 \times 2! \times 8!$
 $+ {}^{10}C_1 \times 1! \times 9!$
 $= 4 \times 10!$
 $= 14\,515\,200$
- (b) Bilangan cara / Number of ways
 $= {}^4C_2 \times {}^6C_3$
 $= 120$
- (c) Bilangan cara / Number of ways
 $= (4 - 1)! \times 2$
 $= 12$

UJIAN 5

KERTAS 1

Bahagian A

1. Jadual di bawah menunjukkan sebahagian taburan binomial untuk 7 percubaan. Kebarangkalian bagi kejayaan ialah p .
- SP** 5.2.1
5.2.2
The table below shows a part of the binomial distribution for 7 trials. The probability for success is p .

$X=r$	0	1	2	3	4	5	6	7
$P(X=r)$		$\frac{416}{729}$					$\frac{11}{243}$	

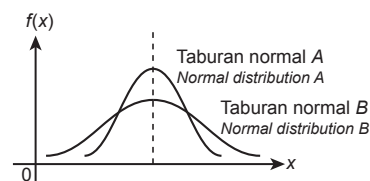
- (a) Cari nilai bagi $P(X = 3) + P(X = 4)$ dalam bentuk pecahan.
Find the value for $P(X = 3) + P(X = 4)$ in fraction form.
[2 markah / 2 marks]
- (b) Seterusnya, tunjukkan bahawa $p = \frac{2}{9q}$, dengan keadaan q ialah kebarangkalian kegagalan bagi setiap percubaan.
Then, show that $p = \frac{2}{9q}$, such that q is the probability of failure for each trial.
[3 markah / 3 marks]
- (c) Jika $p = 2q$, cari nilai p dan nilai q .
If $p = 2q$, find the value of p and of q .
[3 markah / 3 marks]

Jawapan / Answer:

- (a) $P(X = 3) + P(X = 4)$
 $= 1 - \frac{416}{729} - \frac{11}{243}$
 $= \frac{280}{729}$
- (b) ${}^7C_3 p^3 q^4 + {}^7C_4 p^4 q^3 = \frac{280}{729}$
 $35p^3 q^3 (q + p) = \frac{280}{729}$
 $p^3 q^3 = \frac{8}{729} = \left(\frac{2}{9}\right)^3$
 $\therefore pq = \frac{2}{9}$
 $\therefore p = \frac{2}{9q}$
- (c) $p = 2q$
 $(2q)^3 q^3 = \left(\frac{2}{9}\right)^3$
 $8q^6 = \frac{8}{9^3}$
 $q^6 = \frac{1}{3^6}$
 $\therefore q = \frac{1}{3}$
 $p = \frac{2}{3}$

Bahagian B

2. (a) Rajah di bawah menunjukkan dua taburan normal, A dan B, dengan min, $\mu = 400$.
The diagram below shows two normal distributions, A and B, with mean, $\mu = 400$.



Jika varians bagi data X dan Y masing-masing ialah 1 400 dan 2 800, tentukan lengkung yang sesuai untuk mewakili data X dan Y.
If the variance of data X and Y are 1 400 and 2 800 respectively, determine the suitable curve to represent data X and Y.

- [2 markah / 2 marks]
- (b) Jisim seekor itik dari sebuah kebun mengikut taburan normal, dengan min 2.4 kg dan sisihan piawai m kg. Diberi bahawa 16.35% itik dari kebun mempunyai jisim lebih daripada 2.8 kg.
The mass of a duck from a farm follows a normal distribution, with a mean of 2.4 kg and a standard deviation of m kg. Given that 16.35% of ducks from the farm have masses greater than 2.8 kg.
- (i) Cari nilai m .
Find the value of m .
[3 markah / 3 marks]
- (ii) Jika terdapat 1 580 ekor itik, cari bilangan itik yang mempunyai jisim antara 2.2 kg dengan 2.7 kg.
If there are 1 580 ducks, find the number of ducks that have masses between 2.2 kg and 2.7 kg.
[3 markah / 3 marks]

Jawapan / Answer:

- (a) Data X: Taburan normal A
Data X: Normal distribution A
Data Y: Taburan normal B
Data Y: Normal distribution B
- (b) (i) $X \sim N(2.4, m^2)$
 $P(X > 2.8) = 0.1635$
 $P\left(Z > \frac{2.8 - 2.4}{m}\right) = 0.1635$
 $\frac{2.8 - 2.4}{m} = 0.98$
 $0.4 = 0.98m$
 $m = 0.4081$

$$\begin{aligned} \text{(ii)} \quad & P(2.2 < X < 2.7) \\ & = P\left(\frac{2.2 - 2.4}{0.4081} < Z < \frac{2.7 - 2.4}{0.4081}\right) \\ & = 0.4568 \\ & \text{Bilangan itik/Number of ducks} \\ & = 0.4568 \times 1\,580 \\ & = 722 \end{aligned}$$

KERTAS

2

Bahagian A

- SP** 5.3.4
5.3.5
1. Tinggi sekumpulan polis bertaburan normal dengan min 165 cm. Diberi bahawa kebarangkalian seorang polis dipilih secara rawak mempunyai tinggi kurang daripada 164 cm ialah 0.4502.

The heights of a group of policemen are normally distributed with a mean of 165 cm. Given that a policeman chosen at random has a height less than 164 cm is 0.4502.

- (a) Jika 7 orang polis dipilih secara rawak, cari kebarangkalian bahawa
If 7 policemen are chosen at random, find the probability that

(i) tepat 3 orang polis mempunyai tinggi kurang daripada 164 cm.
exactly 3 policemen have heights less than 164 cm.

(ii) selebih-lebihnya 6 orang polis mempunyai tinggi kurang daripada 164 cm.
at most 6 policemen have heights less than 164 cm.

[5 markah / 5 marks]

- (b) Jika seorang polis dipilih secara rawak, cari kebarangkalian bahawa tingginya lebih daripada 168 cm.

If a policeman is chosen at random, find the probability that his height is more than 168 cm.

[4 markah / 4 marks]

Jawapan / Answer:

$$\begin{aligned} \text{(a)} \quad \text{(i)} \quad & X \sim N(165, \sigma^2) \\ & P(X < 164) = 0.4502 \\ & p = 0.4502 \\ & n = 7 \\ & P(X = 3) = {}^7C_3(0.4502)^3(0.5498)^4 \\ & = 0.2918 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & P(X \leq 6) = 1 - P(X = 7) \\ & = 1 - {}^7C_7(0.4502)^7 \\ & = 0.9963 \end{aligned}$$

$$\text{(b)} \quad P(X < 164) = 0.4502$$

$$\begin{aligned} P\left(Z < \frac{164 - 165}{\sigma}\right) &= 0.4502 \\ -\frac{5}{\sigma} &= -0.125 \\ \sigma &= 40 \end{aligned}$$

$$P(X > 168) = P\left(Z > \frac{168 - 164}{40}\right)$$

$$P(Z > 0.1) = 0.4602$$

Bahagian B

- SP** 5.1.1
2. (a) Seramai 80 orang murid menduduki satu ujian diagnostik di sebuah sekolah. Markah yang diperoleh bertaburan normal dengan min 45 dan varians 121.

A total of 80 pupils sat for a diagnostic test at a school. The marks that obtained are normally distributed with a mean of 45 and a variance of 121.

- (i) Murid yang memperoleh markah di antara 45 dengan 65 dikategorikan dalam kumpulan sederhana. Jika seorang murid dipilih secara rawak daripada kumpulan itu, cari kebarangkalian murid itu dalam kumpulan sederhana.

The pupils who scored the marks between 45 and 65 were categorised in the medium group. If a pupil is chosen at random from the group, find the probability that the pupil is in the intermediate group.

- (ii) Diberi bahawa 62% daripada murid dalam kumpulan lulus ujian itu. Hitung markah minimum untuk lulus dalam ujian itu.

Given that 62% of the pupils in the group passed the test. Calculate the minimum mark to pass the test.

[5 markah / 5 marks]

- (b) Kaji selidik juga mendapati bahawa 82% daripada murid di dalam sekolah itu berminat mata pelajaran Matematik. Jika 8 orang murid dipilih secara rawak, cari

The survey also found that 82% of the pupils in the school were interested in Mathematics. If 8 students are chosen at random, find

- (i) sisihan piawai murid yang berminat dalam matematik

the standard deviation of pupils who are interested in Mathematics.

- (ii) kebarangkalian bahawa sekurang-kurangnya 3 orang murid tidak berminat Matematik.

the probability that at least 3 pupils are not interested in Mathematics.

[5 markah / 5 marks]

Jawapan / Answer:

(a) (i) $X \sim N(45, 121)$
 $P(45 < X < 65)$
 $= P\left(0 < z < \frac{65 - 45}{11}\right)$
 $= P(z > 0) - P(z > 1.8182)$
 $0.5 - (0.0351 - 0.0006) = 0.4655$

(ii) $P(X > m) = 0.62$
 $P\left(z > \frac{m - 45}{11}\right) = -0.306$
 $m = 45 - 11(0.306)$
 $= 41.63$

(b) (i) $p = 0.82, q = 0.18, n = 8$
 $\sigma^2 = npq$
 $= 8 \times 0.82 \times 0.18$
 $= 1.1808$
 $\sigma = 1.087$

(ii) $P(Y \geq 3) = 1 - P(Y = 0) - P(Y = 1) - P(Y = 2)$
 $= 1 - {}^8C_0(0.18)^0(0.82)^8 -$
 ${}^8C_1(0.18)^1(0.82)^7 -$
 ${}^8C_2(0.18)^2(0.82)^6$
 $= 0.7242$

3. (a) Kebarangkalian bahawa seorang pelajar menaiki bas ke sekolah ialah p . Satu sampel terdiri daripada 6 pelajar dipilih secara rawak. *The probability that a student takes a school bus to school is p . A sample of 6 students are chosen at random.*

SP
 5.2.2
 5.3.5

(i) Jika kebarangkalian semua menaiki bas ke sekolah ialah 2.44×10^{-4} , cari nilai p . *If the probability that all of them take school bus to school is 2.44×10^{-4} , find the value of p .*

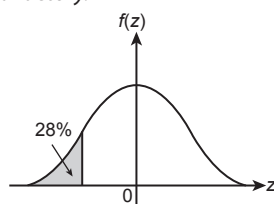
[2 markah / 2 marks]

(ii) Cari kebarangkalian lebih daripada 4 pelajar menaiki bas ke sekolah. *Find the probability that more than 4 students take school bus to school.*

[3 markah / 3 marks]

(b) Rajah menunjukkan satu graf taburan normal piawai yang mewakili isi padu kicap dalam botol yang dihasilkan oleh sebuah kilang.

Diagram shows a standard normal distribution graph which represents the volume of the sauce in bottles produced by a factory.



Diberi min ialah 450 ml dan sisihan piawai ialah 35 ml. Cari

Given that the mean is 450 ml and the standard deviation is 35 ml. Find

(i) nilai v jika peratus isi padu kurang daripada v ialah 28%. *the value of v if the percentage of the volume less than v is 28%.*

[2 markah / 2 marks]

(ii) kebarangkalian bahawa isi padu ialah di antara 455 ml dan 468 ml. *the probability that the volume is between 455 ml and 468 ml.*

[3 markah / 3 marks]

Jawapan / Answer:

(a) $n = 6$

(i) $P(X = 6) = {}^6C_6 p^6 = 2.44 \times 10^{-4}$
 $p = [2.44 \times 10^{-4}]^{\frac{1}{6}}$
 $= 0.25$

(ii) $P(X > 4) = P(X = 5) + P(X = 6)$
 $= {}^6C_5(0.25)^5(0.75) + {}^6C_6(0.25)^6$
 $= 0.0046$

(b) Min ialah 450 ml dan sisihan piawai ialah 35 ml.

Mean is 450 ml and standard deviation is 35 ml

(i) $P(X < v) = 0.28$

$P\left(Z < \frac{v - 450}{35}\right) = 0.28$
 $\frac{v - 450}{35} = -0.583$
 $v = 429.6 \text{ ml}$

(ii) $P(455 < X < 468)$
 $= P\left(\frac{455 - 450}{35} < Z < \frac{468 - 450}{35}\right)$
 $= 0.1397$

UJIAN 6

KERTAS 1

Bahagian A

1. Diberi bahawa $\cos(\alpha + \beta) = \frac{1}{5}$ dan $\sin \alpha \sin \beta = \frac{1}{3}$, cari

6.2.1 *It is given that $\cos(\alpha + \beta) = \frac{1}{5}$ and $\sin \alpha \sin \beta = \frac{1}{3}$, find*

(a) $\cos(\alpha - \beta)$
 $\cos(\alpha - \beta)$

[3 markah / 3 marks]

(b) $\tan \alpha \tan \beta$

[1 markah / 1 mark]

Jawapan / Answer:

(a) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\frac{1}{5} = \cos \alpha \cos \beta - \frac{1}{3}$$

$$\cos \alpha \cos \beta = \frac{8}{15}$$

$$\begin{aligned} \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \frac{8}{15} + \frac{1}{3} = \frac{13}{15} \end{aligned}$$

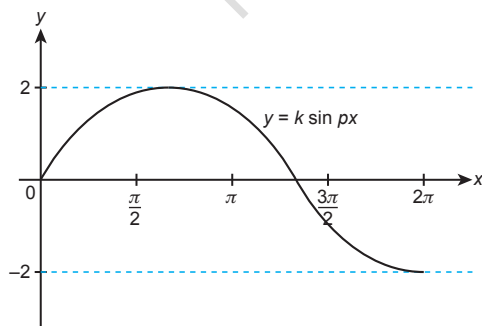
(b) $\tan \alpha \tan \beta = \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}$

$$= \frac{1}{3} \div \frac{8}{15} = \frac{1}{3} \times \frac{15}{8} = \frac{5}{8}$$

Bahagian B

2. (a) Rajah menunjukkan graf $y = k \sin px$ untuk $0 \leq x \leq 2\pi$.

6.2.2 *The diagram shows the graph $y = k \sin px$ for*
6.3.2 $0 \leq x \leq 2\pi$.



(i) Nyatakan nilai k dan nilai p .
State the value of k and of p .

(ii) Pada paksi yang sama, lakarkan satu graf yang sesuai $y = q$ supaya terdapat satu penyelesaian sahaja. Cari nilai yang mungkin bagi q .

On the same axes, sketch a suitable graph $y = q$ so that there is only one solution. Find the possible values of q .

[4 markah / 4 marks]

(b) Diberi $\sin A = p$, dengan keadaan $90^\circ \leq A \leq 180^\circ$. Cari dalam sebutan p .

Given $\sin A = p$, such that $90^\circ \leq A \leq 180^\circ$. Find in terms of p .

(i) $\sec^2 A / \sec^2 A$

(ii) $\cos 4A / \cos 4A$

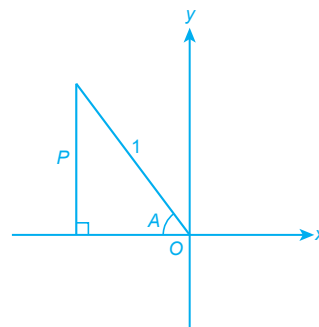
[4 markah / 4 marks]

Jawapan / Answer:

(a) (i) $k = 2, p = \frac{3}{4}$

(ii) $y = q = 2$ atau/or -2

(b)



(i) $\sec^2 A = \frac{1}{\cos^2 A} = \frac{-1}{(1-p^2)}$

(ii) $\begin{aligned} \cos 4A &= 2 \cos 2A - 1 \\ &= 2(1 - 2 \sin^2 A) - 1 \\ &= 1 - 4 \sin^2 A \\ &= 1 - 4p^2 \end{aligned}$

KERTAS 2

Bahagian A

1. (a) Selesaikan persamaan $5 \sin x - 4 \cos^2 x = 2$.
Solve the equation $5 \sin x - 4 \cos^2 x = 2$.

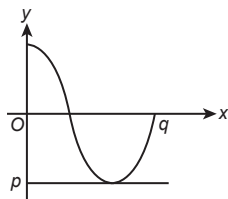
SP
6.3.1
6.3.2

[3 markah / 3 marks]

(b) Rajah di bawah menunjukkan graf $y = \frac{3}{4} \cos 4x$. Nyatakan nilai p dan nilai q .

The diagram below shows a graph $y = \frac{3}{4} \cos 4x$. State the value of p and of q .

[3 markah / 3 marks]



Jawapan / Answer:

(a) $5 \sin x - 4 \cos^2 x = 2$
 $5 \sin x - 4(1 - \sin^2 x) - 2 = 0$
 $5 \sin x - 4 + 4 \sin^2 x - 2 = 0$
 $4 \sin^2 x + 5 \sin x - 6 = 0$
 $(4 \sin x - 3)(\sin x + 2) = 0$
 $\sin x = \frac{3}{4}$ atau/or $\sin x = -2$
 $x = 48^\circ 35', 131^\circ 25'$

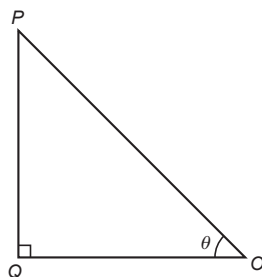
(b) $y = \frac{3}{4} \cos 4x$
 $p = \frac{3}{4}$
 $q = \frac{3}{4} \times \frac{2\pi}{4}$
 $= \frac{3}{8} \pi$

Bahagian B

2. (a) Rajah di bawah menunjukkan sebuah segi tiga tegak POQ.

SP
6.4.1
6.3.2

The diagram below shows a right-angled triangle POQ.



Dengan menggunakan teorem Pythagoras, terbitkan identiti $\sin^2 \theta + \cos^2 \theta = 1$.

By using Pythagoras Theorem, derive the identity $\sin^2 \theta + \cos^2 \theta = 1$.

[3 markah / 3 marks]

(b) Lakarkan graf $y = 1 - \tan 2x$ untuk $0 \leq x \leq \pi$. Seterusnya, dengan paksi yang sama, lakarkan satu garis yang sesuai untuk mencari bilangan penyelesaian bagi persamaan $\tan 2x + \frac{x}{\pi} = 0$. Nyatakan bilangan penyelesaian.

Sketch the graph $y = 1 - \tan 2x$ for $0 \leq x \leq \pi$. Then, use the same axes, sketch a suitable line to find the number

of solutions for the equation $\tan 2x + \frac{x}{\pi} = 0$. State the number of solutions.

[7 markah / 7 marks]

Jawapan / Answer:

(a) $\sin \theta = \frac{PQ}{OP}$ ①

$\cos \theta = \frac{OQ}{OP}$ ②

Menurut teorem Pythagoras,

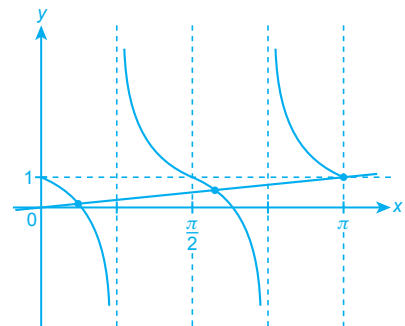
By Pythagoras theorem,

$PQ^2 + OQ^2 = OP^2$

$OP^2 \sin^2 \theta + OP^2 \cos^2 \theta = OP^2$

$OP^2 \sin^2 \theta + OP^2 \cos^2 \theta = OP^2$

Maka/Hence $OP^2 \sin^2 \theta + OP^2 \cos^2 \theta = OP^2$
 $\sin^2 \theta + \cos^2 \theta = 1$



(b) $\tan 2x + \frac{x}{\pi} = 0$

$\frac{x}{\pi} = -\tan 2x$

$y = \frac{x}{\pi}, x = 0, y = 0$ dan/and $x = \pi, y = 1$

3 penyelesaian / 3 solutions

UJIAN 7

KERTAS

2

Bahagian C

SP
7.2.1
1. Sebuah syarikat pengangkutan diminta menghantar 900 orang murid dan 60 buah meja ke sebuah sekolah baharu. Jadual di bawah menunjukkan maklumat tentang muatan kenderaan yang disediakan oleh syarikat.

A transport company was asked to deliver 900 pupils and 60 desks to a new school. The table below shows information about the vehicle load provided by the company.

Jenis Type	Kapasiti duduk Seating capacity	Kapasiti muatan Loading capacity	Kos operasi Operating cost
A	75 orang/ people	8 meja/ tables	RM500
B	120 orang/ people	4 meja/ tables	RM600

(a) Jika x kenderaan A dan y kenderaan B telah digunakan, tulis dua ketaksamaan yang mewakili kekangan masalah tersebut.
If x vehicle A and y vehicle B are used, write two inequalities to represent the constraints of the problem.

[2 markah / 2 marks]

(b) Jika syarikat hanya mempunyai 10 buah kenderaan A dan 8 buah kenderaan B, lukis dan labelkan rantau R yang memuaskan semua kekangan yang diberi.
If the company has 10 type A vehicles and 8 type B vehicles, draw and label the region R that satisfies all the given constraints.

[4 markah / 4 marks]

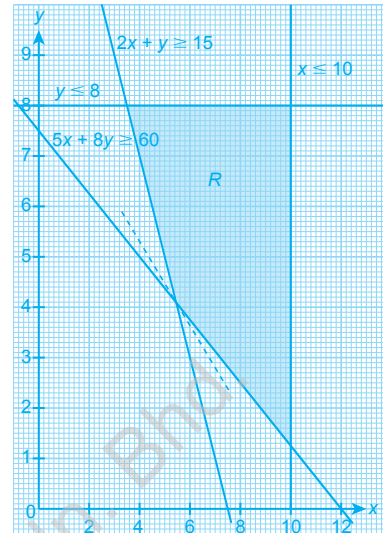
(c) Rumuskan masalah syarikat untuk meminimalkan kos operasi. Cari kos minimumnya.
Formulate the company's problem to minimize its operating cost. Find the minimum cost.

[4 markah / 4 marks]

Jawapan / Answer:

(a) $75x + 120y \geq 900$ atau/ or $5x + 8y \geq 60$
 $8x + 4y \leq 60$ atau/ or $2x + y \leq 15$

(b)



(c) Kos operasi / Operating cost
 $C = 500x + 600y$
Titik optimum / Optimum point (6, 4)
Kos minimum / Minimum cost
 $500(6) + 600(4) = \text{RM}5\,400$

2. Sebuah restoran ingin melengkapkan dewan makan dengan x unit meja P dan y unit meja Q. Pembelian meja tertakluk kepada kekangan yang berikut.

SP
7.2.1

A restaurant wants to equip the dining hall with x units of table P and y units of table Q. The purchase of the tables are limited by the following constraints.

- I Bilangan meja P adalah selebih-lebihnya 10 lebih daripada bilangan meja Q.
The number of table P is at most 10 more than the number of table Q.
- II Jumlah bilangan maksimum meja P dan empat kali meja Q ialah 60.
The maximum number of table P and four times the number of table Q is 60.
- III Jumlah meja Q dan dua kali bilangan meja P adalah sekurang-kurangnya $\frac{2}{3}$ kali jumlah bilangan meja P dan empat kali meja Q.
The total of table Q and two times the number of table P is at least $\frac{2}{3}$ times the total number of table P and four times the number of table Q

- (a) Nyatakan semua ketaksamaan, selain daripada $x \geq 0$ dan $y \geq 0$, yang memuaskan semua kekangan yang diberi.

Write all the inequalities, other than $x \geq 0$ and $y \geq 0$ for all the constraints given.

[3 markah / 3 marks]

- (b) Lukis dan lorekkan rantau R yang memuaskan semua kekangan yang diberi

Draw and shade the region R which satisfies all the constraints given.

[3 markah / 3 marks]

- (c) Daripada graf, cari

From the graph, find

- (i) julat bilangan meja Q jika 10 buah meja P dibeli.

the range of the number of table Q if 10 table P are bought.

- (ii) bilangan maksimum pelanggan yang boleh muat pada satu masa tertentu jika meja jenis P dan jenis Q masing-masing boleh memuat 8 dan 10 pelanggan.

the maximum number of customers that can use the tables at a specific time if table P and table Q can accommodate 8 and 10 customers respectively.

[4 markah / 4 marks]

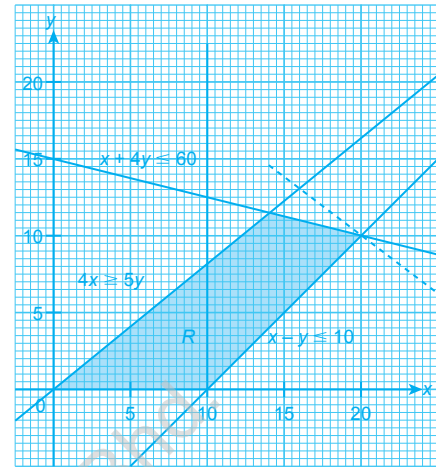
Jawapan / Answer:

I: $x - y \leq 10$

II: $x + 4y \leq 60$,

III: $2x + y \geq \frac{2}{3}(x + 4y)$
 $4x \geq 5y$

(b)



- (c) (i) $0 \leq y \leq 8$

(ii) $k = 8x + 10y$

Titik optimum (20, 10)

Bilangan maksimum pelanggan

Maximum number of customers

$$= 8(20) + 10(10)$$

$$= 260$$

UJIAN 8

KERTAS

2

Bahagian C

- SP** 8.4.1
1. Suatu zarah bergerak pada satu garis lurus dengan halaju, $v \text{ m s}^{-1}$, yang diberi oleh $v = 12t - 5 - 3t^2$, dengan keadaan t ialah masa, dalam saat, selepas melalui O . Hitung [Anggapkan gerakan ke arah kanan sebagai positif]

A particle moves along a straight line with a velocity, $v \text{ m s}^{-1}$, that is given by $v = 12t - 5 - 3t^2$, such that t is the time, in seconds, after passing O . Calculate [Assume motion to the right is positive]

- (a) halaju maksimum, dalam m s^{-1} , bagi zarah itu.
the maximum velocity, in m s^{-1} , of the particle. [2 markah / 2 marks]
- (b) sesaran, dalam m , zarah itu pada ketika $t = 2 \text{ s}$.
the displacement, in m , of the particle when $t = 2 \text{ s}$. [3 markah / 3 marks]
- (c) masa yang mungkin, dalam saat, apabila zarah itu melalui O sekali lagi.
the possible times, in seconds, when the particle passes O again. [3 markah / 3 marks]
- (d) halaju, dalam m s^{-1} , apabila pecutan zarah itu ialah -6 m s^{-2} .
the velocity, in m s^{-1} , when the acceleration of the particle is -6 m s^{-2} . [2 markah / 2 marks]

Jawapan / Answer:

(a) $v = 12t - 5 - 3t^2$

$$\frac{dv}{dt} = 12 - 6t$$

$$V_{\text{maksimum / maximum}} = \frac{dv}{dt} = 0$$

$$12 - 6t = 0$$

$$t = 2 \text{ s}$$

$$v = 12(2) - 5 - 3(2)^2$$

$$= 24 - 5 - 12$$

$$= 7 \text{ m s}^{-1}$$

(b) $s = \int 12t - 5 - 3t^2 dt$

$$= 6t^2 - 5t - t^3 + c$$

Apabila / When $t = 0, s = 0, c = 0$

Apabila / When $t = 2,$

$$s = 6(2)^2 - 5(2) - 2^3$$

$$= 24 - 10 - 8$$

$$= 6 \text{ m}$$

(c) Apabila / When $s = 0$

$$6t^2 - 5t - t^3 = 0$$

$$t(6t - 5 - t^2) = 0$$

$$t(t - 5)(1 - t) = 0$$

$$t = 1 \text{ s atau / or } t = 5 \text{ s}$$

(d) $a = \frac{dv}{dt} = 12 - 6t$

Apabila / When $a = -6$

$$12 - 6t = -6$$

$$t = 3 \text{ s}$$

$$v = 12(3) - 5 - 3(3)^2$$

$$= 4 \text{ m s}^{-1} \text{ (ke arah kanan / to the right)}$$

- SP** 8.4.1
2. Suatu zarah bergerak pada satu garis lurus, Pecutannya, $a \text{ m s}^{-2}$, diberi oleh $a = 6 - t$, dengan keadaan t ialah masa, dalam saat, selepas melalui titik tetap O . Halaju awal zarah itu ialah 14 m s^{-1} dan zarah itu berhenti pada titik A . Hitung [Anggapkan gerakan ke arah kanan sebagai positif]

A particle moves along a straight line. Its acceleration, $a \text{ m s}^{-2}$, is given by $a = 6 - t$, such that t is the time, in seconds, after passing a fixed point O . The initial velocity of the particle is 14 m s^{-1} and the particle stops at the point A . Calculate [Assume motion to the right is positive]

- (a) masa, dalam saat, yang diambil oleh zarah itu untuk sampai ke titik A .
the time, in seconds, taken by the particle to reach to the point A .

[4 markah / 4 marks]

- (b) jarak OA , dalam m .
the distance of OA , in m .

[3 markah / 3 marks]

- (c) halaju maksimum, dalam m s^{-1} , sepanjang pergerakannya dari O ke A .
the maximum velocity, in m s^{-1} , throughout its journey from O to A .

[3 markah / 3 marks]

Jawapan / Answer:

(a) $a = 6 - t$

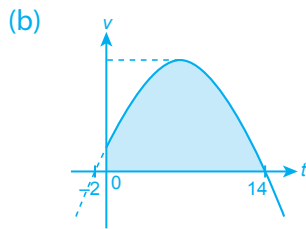
$$v = \int 6 - t dt$$

$$= 6t - \frac{t^2}{2} + c$$

Apabila / When $t = 0, v = 14$

$$v = 6t - \frac{t^2}{2} + 14$$

$$\begin{aligned} \text{Apabila / When } v &= 0 \\ 0 &= 12t - t^2 + 28 \\ 0 &= (14 - t)(2 + t) \\ t &= 14 \text{ atau / or } t = -2 \\ \therefore t &= 14 \text{ s} \end{aligned}$$



$$\begin{aligned} \text{Jarak OA / Distance of OA} &= \int_0^{14} 6t - \frac{t^2}{2} + 14dt \\ &= \left[3t^2 - \frac{t^3}{6} + 14t \right]_0^{14} \\ &= 3(14)^2 - \frac{14^3}{6} + 14(14) - 0 \\ &= 326.67 \text{ m} \end{aligned}$$

(c) $v_{\text{maksimum / maximum}} = a = 0$

$$\begin{aligned} 6 - t &= 0 \\ t &= 6 \\ v &= 6(6) - \frac{(6)^2}{2} + 14 \\ &= 32 \text{ m s}^{-1} \end{aligned}$$

- 3.** Suatu zarah bergerak pada satu garis lurus dengan halaju $v \text{ m s}^{-1}$, yang diberi oleh $v = t^2 - 10t + 24$, dengan keadaan t ialah masa, dalam saat, selepas melalui titik tetap O. Hitung [Anggapkan gerakan ke arah kanan sebagai positif]

SP
8.4.1

A particle moves along a straight line with a velocity of $v \text{ m s}^{-1}$, that is given by $v = t^2 - 10t + 24$, such that t is the time, in seconds, after passing through a fixed point O. Calculate [Assume motion to the right is positive]

- (a) julat masa, dalam saat, apabila zarah itu bergerak ke kiri O.
the time interval, in seconds, when the particle moves to the left of the O.
- [2 markah / 2 marks]
- (b) halaju tertinggi, dalam m s^{-1} , bagi zarah itu untuk $0 \leq t \leq 6$.
the highest velocity, in m s^{-1} , of the particle for $0 \leq t \leq 6$.
- [2 markah / 2 marks]
- (c) jarak, dalam m, yang dilalui oleh zarah itu dalam saat ketiga.
the distance, in m, travelled by the particle in the third second.
- [3 markah / 3 marks]
- (d) sesaran dari O apabila pecutan zarah itu ialah sifar.
the displacement from O when the acceleration of the particle is zero.

[3 markah / 3 marks]

Jawapan / Answer:

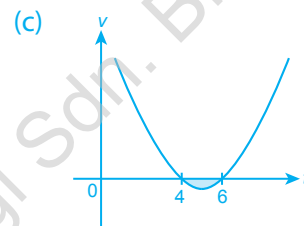
(a) $v = t^2 - 10t + 24$
Apabila zarah bergerak ke kiri, $v < 0$.
When the particle moves to the left, $v < 0$.

$$\begin{aligned} t^2 - 10t + 24 &< 0 \\ (t - 4)(t - 6) &< 0 \\ t &< 4 \text{ atau / or } t < 6 \\ \text{Julat masa / Time interval} \\ &4 < t < 6 \end{aligned}$$

(b) $v_{\text{maksimum / maximum}} = \frac{dv}{dt} = 0$

$$\begin{aligned} \frac{dv}{dt} &= 2t - 10 = 0 \\ t &= 5 \text{ s} \\ v &= 5^2 - 10(5) + 24 \\ &= -1 \text{ m s}^{-1} \end{aligned}$$

Apabila / When $t = 0$, $v = 24 \text{ m s}^{-1}$
Halaju tertinggi / The highest velocity
 $= 24 \text{ m s}^{-1}$



$$\begin{aligned} \text{Jarak / Distance } s &= \int_4^6 (t^2 - 10t + 24) dt \\ &= \left[\frac{t^3}{3} - 5t^2 + 24t \right]_4^6 \\ &= 5\frac{1}{3} \text{ m} \end{aligned}$$

(d) $s = \frac{t^3}{3} - 5t^2 + 24t$

Apabila / When $t = 5$, $s = \frac{5^3}{3} - 5(5)^2 + 24(5)$
 $= 36\frac{2}{3} \text{ m}$

- 4.** Suatu zarah P bergerak pada satu garis lurus dan melalui satu titik tetap O, dengan halaju -36 m s^{-1} . Pecutannya diberi oleh $a_p = 6t - 12 \text{ m s}^{-2}$, dengan keadaan t ialah masa, dalam saat, selepas melalui O. Pada masa yang sama, satu lagi zarah Q dengan jaraknya ialah 20 m ke kanan O bergerak menuju ke arah O dengan halaju tetap -3 m s^{-1} . [Anggapkan gerakan ke arah kanan sebagai positif]
- A particle P moves along a straight line and passes a fixed point O, with a velocity of -36 m s^{-1} . Its acceleration is given by $a_p = 6t - 12 \text{ m s}^{-2}$, such that t is the time taken, in seconds, after passing through O. At the same time, another particle Q is 20 m to the right of O moves towards O with a constant velocity of -3 m s^{-1} .*
- [Assume motion to the right is positive]

SP
8.4.1

- (a) Cari julat masa, dalam saat, apabila zarah P bergerak ke arah kanan.
Find the range of time, in seconds, when particle P is moving to the right.
 [3 markah / 3 marks]
- (b) Cari jarak, dalam m, antara zarah P dengan zarah Q apabila zarah P berhenti seketika.
Find the distance, in m, between particle P and particle Q when particle P stops momentarily.
 [3 markah / 3 marks]
- (c) Cari halaju maksimum, dalam m s^{-1} , bagi zarah P .
Find the maximum velocity, in m s^{-1} , of particle P .
 [2 markah / 2 marks]
- (d) Hitung masa apabila halaju P ialah sama dengan halaju Q . Beri jawapan anda betul kepada 2 tempat perpuluhan.
Calculate the time when the velocity of P is the same as the velocity of Q . Give your answer correct to 2 decimal places.
 [2 markah / 2 marks]

Jawapan / Answer:

- (a) $a_p = 6t - 12$
 $v_p = \int (6t - 12) dt$
 $= 3t^2 - 12t + c$
 Apabila / When $t = 0$, $v_p = -36$, $c = -36$
 Oleh kerana zarah P bergerak ke kanan,
 $v_p > 0$
 Since the particle P is moving to the right, $v_p > 0$
 $3t^2 - 12t - 36 > 0$
 $t^2 - 4t - 12 > 0$
 $(t + 2)(t - 6) > 0$
 $t > -2$ atau / or $t > 6$
 Zarah P bergerak ke kanan, apabila $t > 6$.
 Particle P is moving to the right when $t > 6$.

- (b) Bagi zarah P / For particle P ,
 Apabila / When $t = 6$,
 $v_p = 3t^2 - 12t - 36 = 0$
 $s_p = \int (3t^2 - 12t - 36) dt$
 $= t^3 - 6t^2 - 36t + c$
 Apabila / When $t = 0$, $s_p = 0$, $c = 0$
 $s_p = t^3 - 6t^2 - 36t$
 Apabila / When $t = 6$,
 $s_p = 6^3 - 6(6)^2 - 36(6)$
 $= -216 \text{ m}$
 $v_Q = -3$
 $s_Q = -3t + c$
 Bagi zarah Q , apabila $t = 0$, jadi $s_Q = -3t + 20$
 For particle Q , when $t = 0$, $s_Q = 20$, then $s_Q = -3t + 20$
 Apabila / When $t = 6$, $s_Q = -2(6) + 20 = 2 \text{ m}$
 Jarak antara zarah P dengan Q .
 Distance between particle P and Q .
 $= 216 + 2$
 $= 218 \text{ m}$

- (c) $a_p = 6t - 12 = 0$, $t = 2$
 Apabila / When $t = 2$,
 $v_p = 3(2)^2 - 12(2) - 36$
 $= -48 \text{ m s}^{-1}$

- (d) $v_p = v_Q$
 $v_p = 3t^2 - 12t - 36$
 $v_q = -3$
 $3t^2 - 12t - 36 = -3$
 $t^2 - 4t - 11 = 0$
 $t = \frac{4 \pm \sqrt{16 + 44}}{2}$
 $= 5.87 \text{ s}$